Mathematical model of a closed-loop oscillating heat pipe with check valves

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Abstract

The heat transfer characteristics of a closed-looped oscillating heat pipe with check valves (CLOHP/CV) were modeled using the explicit finite element method. The parameters affecting the heat transfer of a CLOHP/CV were analytically investigated. The principles and theories of basic governing equations and a finite difference scheme were applied to evaluate the heat transfer rate. The predicted heat transfer rate, obtained from the model, was compared to the existing experimental data. The effect of evaporator length, inner diameter and working fluid on the performance of a CLOHP/CV were also investigated.

Keywords: Closed-looped oscillating heat-pipe, Check valves, Mathematical modeling

Introduction

The oscillating heat pipe is to divided three types. The first is a closed-end oscillating heat-pipe (CEOHP), which is closed at both ends. The second is a closed-loope oscillating heat-pipe (CLOHP), which is connected at both ends of the tube to form a closed loop. The last is a closed-loope oscillating heat-pipe with a check valves (CLOHP/CV), as show in Fig. 1. The heat transfer of these OHPs occur because of the self-sustaining oacilatory flow by a vapour-liquid circulation cycle between the heating and cooling sections: latent heat is transferred.Under normal operating conditions, the liquid and vapour are effectively separated into two parts with the liquid in the cooling regions and the vapour in the heating regions. The liquid forms U-shaped columns in individual turns and their oscillations form waves. When the amplitude of oscillatory flow is sufficient and the heat-transfer area is not included in the waves effective working fluid supply to the heat-transfer area cannot be obtained and heat-transfer cannot be maintained. This operating limit is peculiar to oscillating heat-pipes. However, the installation of check valves in the closed-loop eliminates this operating limit whereby a single-direction flow is imposed and the heat-transfer area is not restricted by the amplitude of the oscillatory flow. Miyazaki et al. studied the

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oscillating heat-pipe including a check valve:under normal operating conditions.It was found that the check valve capable control the flow direction of working fluid. Pipatpaiboon et al. studied the effect of inclination angle, working fluid and number of check valves on the characteristics of heat-transfer in a closed-looped oscillating heat-pipe with check valves (CLOHP/CVs). Experiments were conducted to find out their effects on the heat-transfer rates of a copper CLOHP/CV with an inner diameter of 2.03 mm. The lengths of the evaporator, adiabatic and condenser sections were 5 cm. The total lengths of the CLOHP/CV were 15 m with 40 meandering turns. R123, ethanol and water were used as the working fluids with a filling ratio of 50%. The number of check valves were 2, 5, 8 and 10. It was fould from the experiment that the values of the maximum heat-transfer rate (Q_{max}) of the CLOHP/CV for r123, ethanol and water were 872, 635 and 585 W, respectively. The angle at which Q_{max} occurs was 90°, 80° and 0°
respectively. It can be seen that the check valves affected the heat-transfer characteristics of the CLOHP/CV such that with two check valves for r123 and ethanol, the values of Q_{max} /Q_0 are 1.54 and 1.8, respectively.
However, the five check valve system gave Q_{max} /Q_0 as unity. The systematic experimental data was collected from the research of Rittidech et al. studied the heat transfer characteristics of closed-looped oscillating heat pipe with check valves. The CLOHP/CV used employed a copper tube with inner diameters of 1.77 and 2.03 mm. The evaporator, adiabatic and condenser sections were equal to 50, 100 and 150 mm. Water ethanol and R123 were used as the working fluid with a filling ratio of 50% of total volume. The number of turns was 40. The ratio of check valves were 20, 8, 5 and 4. It was found that a correlation for predicting the heat transfer rate at vertical position was proposed for a CLOHP/CV. Dobson et al. studied the lumped parameter analysis of a closed and open oscillatory heat-pipe. They assumed that the effect of surface tension was negligible and that no heat transfer occurred between the liquid and its surroundings. Wong et al. studied a theoretical model of a OHP by using a formulation based on the Lagrangian method. It was shown that the pressure and velocity varied with time. Shafii et al. presented an analytical model for a symmetric CEOHP and a CLOHP with two turns base on the explicit finite element method. It was concluded that the heat-transfer in the OHP was due to sensible heat and the number of liquid slugs were always reduced to that of the number of heating sections no matter how many initial liquid slugs there were initially in the OHP. Sakulchangsatjatai et al. presented

operation modeling of closed-end and closed-loop oscillating heat-pipe at normal operating condition with number of turns must be more

Theoretical model

Check valve are used to control direction of working fluid. It was configuration consisted of four parts such as case, ball stopper, conical valve seat and ball, show in Fig.2. The mathematic model was established from governing equations for the vapor and liquid phase. Schematic diagrams of CLOHP/CV are show in Fig. 3. The condenser section of the heat-pipe is above evaporator section. The heat-transfer in CLOHP/CV can be predicted by solving mass, momentum and energy and equations for each liquid slug and vapor plug. The appropriate maximum inner diameter of OHP can be calculated by using equation of Maezawa et al.

$$d_{max} \le 2\sqrt{\frac{\sigma}{\rho_l g}} \qquad (1)$$

The prediction of heat-transfer in a CLOHP/CV the following assumed are made:

1. Evaporative and condensation and overall heat transfer coefficients are assumed to be constants.

2. The liquid slugs assumed to be incompressible and the vapor plugs assumed to be have as ideal gas.

3. The amount of liquid slugs corresponds to the number of turns. The length of the liquid slug along the centerline of the tube is dependent on the filling ratio.

4. The pressure losses at the bends can be neglected so it can be assumed that the CLOHP/CV are straight.

5. The layer of liquid film can be neglected.

6. The pressure drop at check valve can be neglected so it very small.

7. The inside phenomena is annular flow.

3. Basic governing equations

Continuity Equations: The modes of the mass transfer through a control volume of vapor bubble depend on the position within a vapor plug. The mass of the vapor plug and liquid slug may be written as

$$m_{vi} = \rho_{vi}\pi L_{vi} d^2 / 4 \qquad (2)$$

$$m_{li} = \rho_{li}\pi L_{li} d^2 / 4 \qquad (3)$$

The rate of change of mass of the vapor plug and liquid slug can be found from the following equations:

$$\frac{dm_{vi}}{dt} = \dot{m}_{in,vi} \quad \dot{m}_{out,vi} \qquad (4)$$

$$\frac{dm_{li}}{dt} = \dot{m}_{in,li} \quad \dot{m}_{out,li} \qquad (5)$$

It is assumed that the rate of change of mass of liquid slug is equal to the average changes in mass of its adjacent vapor plugs.

$$\frac{dm_{li}}{dt} = \frac{1}{2} \quad \frac{dm_{vi}}{dt} + \frac{dm_{v(i+1)}}{dt} \qquad (6)$$

Where $\dot{m}_{in,vi}$ is the mass flow rate due to the evaporation into the *i*th vapor plug and $\dot{m}_{out,vi}$ is the mass flow rate due to the condensation from the vapor plug for *i*th vapor plug, the *i*th vapor plug can be expressed as

$$\dot{m}_{in,vi} = U_e \pi d Le_{vi} (T_h \quad T_{vi}) / h_{fg} \qquad (7)$$

$$\dot{m}_{out,vi} = U_c \pi d Lc_{vi} (T_{vi} \quad T_c) / h_{fg} \qquad (8)$$

Momentum Equations: the momentum equation for *i*th liquid slug is

$$\frac{dm_{li} v_{li}}{dt} = (P_{vi} \quad P_{v(i+1)})A \quad \pi d L_{li} \tau \quad (\ 1)^n m_{li} g \quad P_{checkvalves} \qquad (9)$$

The pressure of check valve can be neglected so it very small, then it can be shown that equation 9 becomes:

$$\frac{dm_{li} \upsilon_{li}}{dt} = (P_{vi} \quad P_{v(i+1)})A \quad \pi d L_{li} \tau_{li} \quad m_{li} g \qquad (10)$$

Since it is assumed that the OHP is a straight tube, gravity has different signs at different locations. In this model, the gravity force can be determined by measuring the difference in level between both ends of the liquid slug. τ is the shear stress acting between *i*th liquid slug and the wall tube and can be determined from

$$\tau_{li} = \frac{1}{2} C_{li} \rho_l \upsilon_{li}^2 \qquad (11)$$

Energy Equations: The energy equation of a vapor plug is

$$m_{vi} C_V \frac{dT_{vi}}{dt} = (\dot{m}in,vi \quad \dot{m}out,vi) RT_{vi} \quad P_{vi} A \frac{dX_{vi}}{dt} \qquad (12)$$

Heat transfer in the OHP is defined as the total heat transferred from the evaporator sections to the condenser sections. It can be divided into two parts, first the heat transfer is due to the evaporation and condensation of the working fluid at vapor phase. The heat transferred into and out of the vapor plug is

$$\dot{Q}_{in,vi} = \dot{m}_{in,vi} h_{fg} \qquad (13)$$

$$\dot{Q}_{out,vi} = \dot{m}_{out,vi} h_{fg} \qquad (14)$$

second, heat transfer between the tube wall and the liquid slugs is obtained by solving the energy equation for a liquid slug

$$\frac{1}{\alpha_{li}} \frac{dT_{li}}{dt} = \frac{d^2 T_{li}}{dX^2} \quad \frac{h_{li} \pi d}{k_l A} (T_{li} \quad T_w) \qquad (15)$$

The boundary conditions of the liquid slug temperature is

$$X = X_{re,i}; \qquad T_{li} = T_{vi}$$
$$X = X_{le,(i+1);} \qquad T_{li} = T_{v(i+1)} \qquad (16)$$

Heat-transferrate: The heat transfer of CLOHP/CV is to define as the total heat transfer from the evaporator sections to the condenser sections. It can be calculated by equations (16) and (17).

$$Q_{in.vi} = \dot{m}_{in.vi} h_{fg} \qquad (17)$$

$$Q_{out.vi} = \dot{m}_{out.vi} h_{fg} \qquad (18)$$

Heat transfer into and out of liquid slug as

$T_{li} > T_W$ when

$$Q_{in.li} = \int_{x_{re.i}}^{x_{le.(i+1)}} \pi D h_x (T_{li.x} - T_w) dx \qquad (19)$$

$T_{li} < T_w$ when

$$Q_{out.li} = \int_{x_{re.i}}^{x_{le.(i+1)}} \pi D h_x (T_w - T_{li.x}) dx$$

The total heat transfer can be defined as

$$Q_{total.in} = \sum_{i=1}^{N} Q_{in.vi} + \sum_{i=1}^{N-1} Q_{in.li} \quad (20)$$

$$Q_{total.out} = \sum_{i=1}^{N} Q_{out.vi} + \sum_{i=1}^{N-1} Q_{out.li}$$

The governing equation is differential. It can be solved using an implicit finite difference scheme. The time step was used in all numerical simulations as 10^{-4} s. The new values at time step (t+⇆t) can be determined from the old values at time t by

$$m_{vi}^{new} = m_{vi} + (\dot{m}_{in.vi} \quad \dot{m}_{out.vi})\Delta t \quad (21)$$

$$T_{vi}^{new} = T_{vi} + \frac{(\dot{m}_{in.vi} \quad \dot{m}_{out.vi}) C_P T_{vi} \Delta t \quad P_{vi} A \Delta x_{vi}}{m_{vi} C_V} \quad (22)$$

$$P_{vi}^{new} = \frac{m_{vi} R T_{vi}}{V_{vi}} \quad (23)$$

$$m_{li}^{new} = m_{li} + \frac{1}{2}\left[(\dot{m}_{in.vi} - \dot{m}_{in.vi}) + (\dot{m}_{in.v(i+1)} - \dot{m}_{in.v(i+1)})\right]\Delta t \quad (24)$$

$$m_{li}^{new} v_{li}^{new} = m_{li} v_{li} + \left[(P_{vi} - P_{v(i+1)})A - \pi d L_{li} \tau - (-1)^n m_{li} g - P_{checkvalve}\right]\Delta t \quad (25)$$

Numerical Solutions: The new valves replaced the old valves and the procedure was repeated for the next time step. The locations of the vapor bubble and liquid slug of CLOHP/CV can be known by the location of their ends. CLOHP/CV the new position of each vapor plug can be defined as

$$x_{re,i}^{new} = x_{re,i} + v_{li}\Delta t$$
$$x_{le,i}^{new} = x_{le,i} + v_{l(i\ \ 1)}\Delta t \quad (26)$$

$$x_{re,i}^{new} = x_{re,i}^{new} \quad L; \qquad x_{re,i}^{new} > L$$
$$x_{le,i}^{new} = x_{le,i}^{new} \quad L; \qquad x_{le,i}^{new} > L \quad (27)$$

$$x_{re,i}^{new} = x_{re,i}^{new} + L; \qquad x_{re,i}^{new} < 0$$
$$x_{le,i}^{new} = x_{le,i}^{new} + L; \qquad x_{le,i}^{new} < 0 \quad (28)$$

Where the length difference in Eq. (10) can be defined as

$$\Delta x_{re,i} = x_{re,i}^{new} - x_{re,i}$$
$$\Delta x_{le,i} = x_{le,i}^{new} - x_{le,i} \quad (29)$$

$$\Delta x_{vi} = \Delta x_{re,i} - \Delta x_{le,i} \quad (30)$$

Hence the flow chart main calculation is as follow Fig. 4.

4. Resultsand discussion

Table 1 shows the results from the model and experiment.

4.1.Comparison with the existing model and experimental data

In order to compared the presented mathematical model with the existing systematic data of Rittidech et al. Their experimental conditions are as follows:

- Inner diameter tubes = 1.77 or 2.03 mm.
- Evaporator, adiabatic and condenser sections are of equal lengths = 50, 100 and 150 mm.
- Number of turn = 40.
- Filling ratio = 50%.
- An evaporator temperature = 80 ^{0}C.
- A condenser temperature = 20 ^{0}C.

- Number of check valve = 1.
- Weight of ball = 0.25 g.

Fig. 5 shows the comparison of the predicted heat flux from the model with the experimental data. It was found that the heat fluxes from prediction can loosely fit with those of the heat flux from experiment. Because, it is necessary to add more assumptions for the formulation of the mathematical model in order to correspond with the real operational condition. However, all data trends to follow the trend line of linear function with implies that the present model in very near future. Corrective expression of heat flux of CLOHP/CV can firstly be found from this comparison as

$$q_{predict} = 1.43xq_{model} -0.035Le \qquad (31)$$

after applying such corrective expressions the exact heat flux can be calculated from the heat flux of prediction of the model. The comparison of heat fluxes from the prediction and the heat fluxes from the experimental data is, again, shown in Fig. 6. It can be seen that all data trends to distribute in the exact line with the standard deviation of ±30%.

4.2. Effect of evaporator length on heat-transfer rate

Fig. 7 shows the comparison of the predicted heat-transfer rate from the model with the experimental data of Rittidech et al. In this experiment,The CLOHP/CV with inner diameter 2.03mm andR123 was used as the working fluid.It can be seen that, if the evaporator length increases, the heat-transfer rate decreases. This may be because when the evaporator is long, the amplitude of motion of vapor and liquid slugs will increase but the frequency will decrease. Thus the heat flux is low.

It can be conclude that, from the effect of the evaporator length, the present model is reliable and its prediction is systematic.

4.3. Effect of inner diameter on heat-transfer rate

Fig. 7 shows the comparison of the predicted heat-transfer rate from the model with the experimental data [5]. In this experiment, The CLOHP/CV with R123 was used as the working fluid and evaporator length of 50 mm. It can be seen that, when the inner diameter increases the heat-transfer rate also rises because if the high inner diameter the boiling phenomenon approaches pool boiling in which its heat transfer coefficient is higher than that of the boiling inside a confined channel with a small diameter. Furthermore, the rate of heat flux from the experiment is more than that of prediction. This might be because Maezawa et al., found that the inner diameter of tube must be lower than critical inner diameter in Eq. (1) in order to obtain the slug train pattern of working fluid in the tube. Therefore, in this experiment the inner diameter is suitable for pattern form of working fluid in the tube.

It can be conclude that, from the effect of the inner diameter, the present model is reliable and its prediction is systematic.

4.4. Effect of working fluid on heat-transfer rate

Fig. 8 shows the comparison of the predicted heat-transfer rate from the model with the experimental data of Rittidech et al. In this experiment, The CLOHP/CV with inner diameter 2.03 mm and evaporator length of 50 mm.It can be seen that, the working fluid can be arranged from the minimum heat flux to the maximum heat flux as water, ethanol and R123. The latent heat decreases, the phase change of working fluid in the condenser and evaporator sections becomes increasingly active. Thus increased heat can be transferred by the working fluid. Furthermore, it can be seen that the trend of heat flux from the experiment is more than that of prediction. This is because the heat transfer rate calculated from model results of the vaporization latent heat has an effect on mass transfer of annular slug and the saturated working pressure has and effect on the velocity of annular slug. In addition, in this experiment the inside phenomena of CLOHP/CV may be is vapor slug and liquid slug. These phenomena is increases transfer heat more than annular phenomena. So the heat flux of the experiment is more than model.

It can be conclude that, from the effect of working fluid, the present model is reliable and its prediction is systematic.

5. Conclusions

The mathematical model to predict the heat flux of CLOHP/CV has been established. From several procedures of comparison, it is found that:

-When compare the heat transfer rate of CLOHP/CV as predict and experiment results are same direction and error interval ±30%.

- The obtained mathematical model can be reliably used.

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Nomenclature

A	area, m^2
C	friction coefficient
C_v	specific heat, J/kgK
d	diameter, m
d	ordinary differential
g	gravitational constant, m/s^2

h heat transfer coefficient, W/m^2 K

h_{fg} latent heat of vaporization, J/kg

k thermal conductivity, W/mK

L length, m

La length of adiabatic section, m

Lc length of condenser section, m

Le length of evaporator section, m

m mass, kg

n number of tube

N number of parallel plug

Q heat transfer rate, W

P pressure, N/m^2

Pr Prandtl number = $c_p\mu/k$

R specific gas constant, J/kgK

Re Reynolds number = $\rho vd/\mu$

T temperature, K

t time, s

U overall heat transfer coefficient, W/m^2

V volume, m^3

v velocity, m/s

X distance, m

Greek letters

α thermal diffusivity, m^2/s

⇆ difference

μ dynamic viscosity, kg/ms

σ surface tension, N/m

ρ density, kg/m^3

τ shear stress, N/m^2

Subscripts

a adiabatic

c cold, condenser

e evaporator

exp experimental value

f friction

g gas

h hot

in inlet

l liquid

li ith liquid plug

le left end

out outlet

prd prediction value

re right end

vi ith vapor plug

w wall tube

X horizontal coordinate

Superscript

new new value

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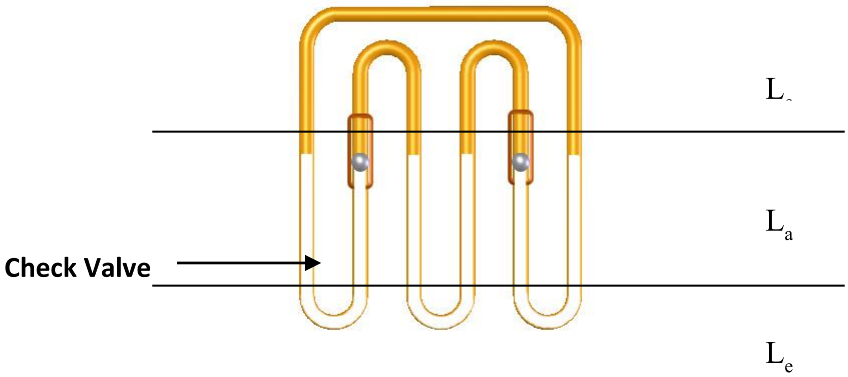

Fig. 1. CLOHP/CV.

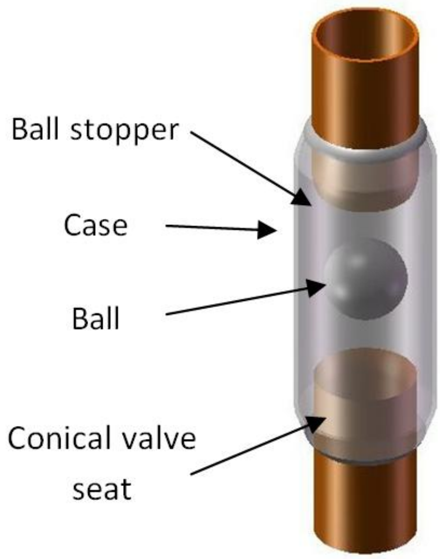

Fig. 2. Prototype of check valve.

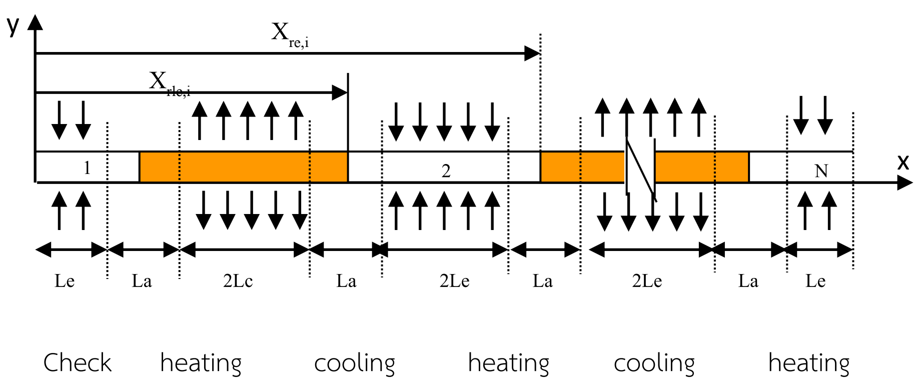

Fig. 3. Assumed CLOHP/CV to be straight tube.

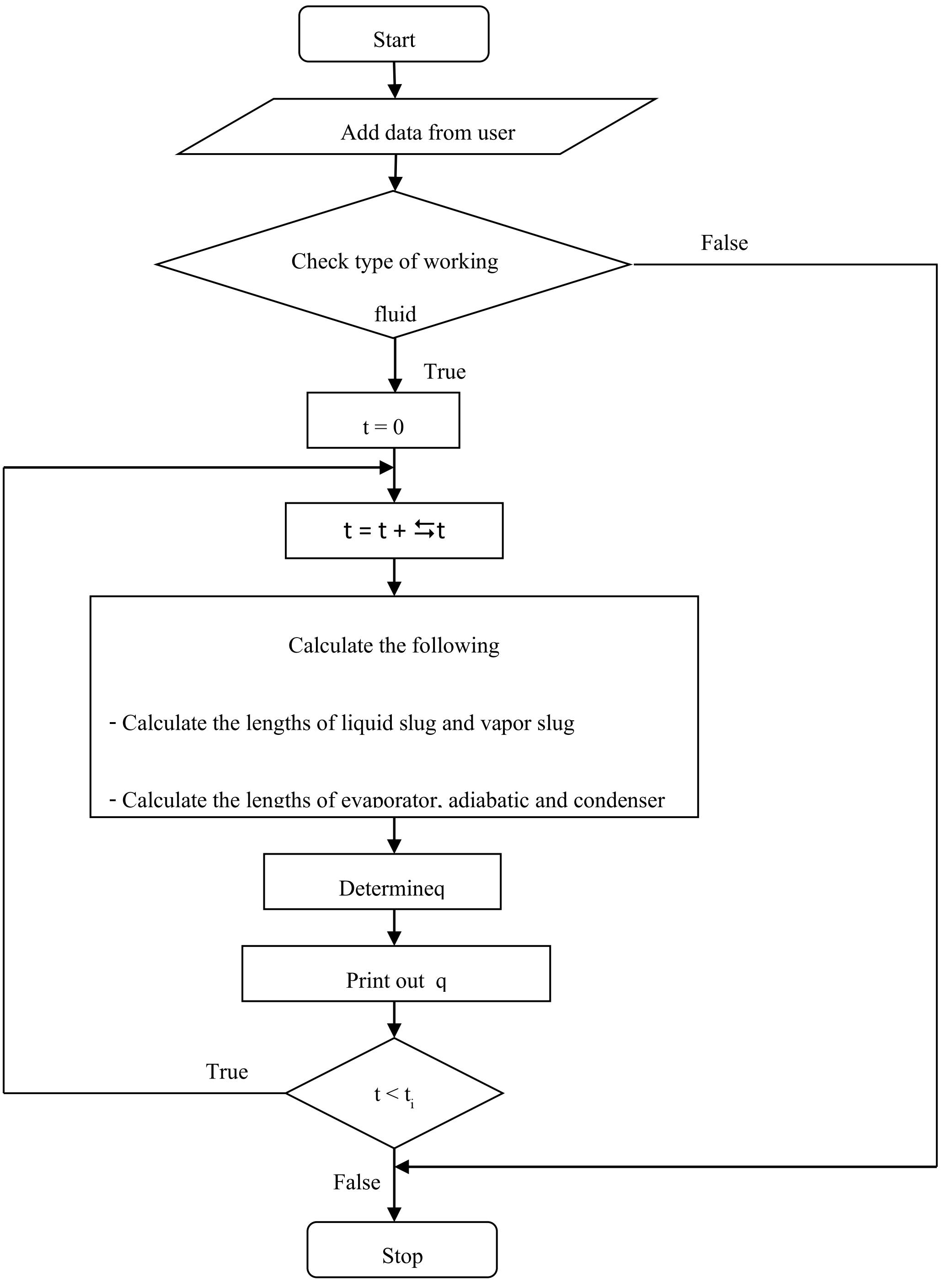

Fig. 4. Calculation flow chart main.

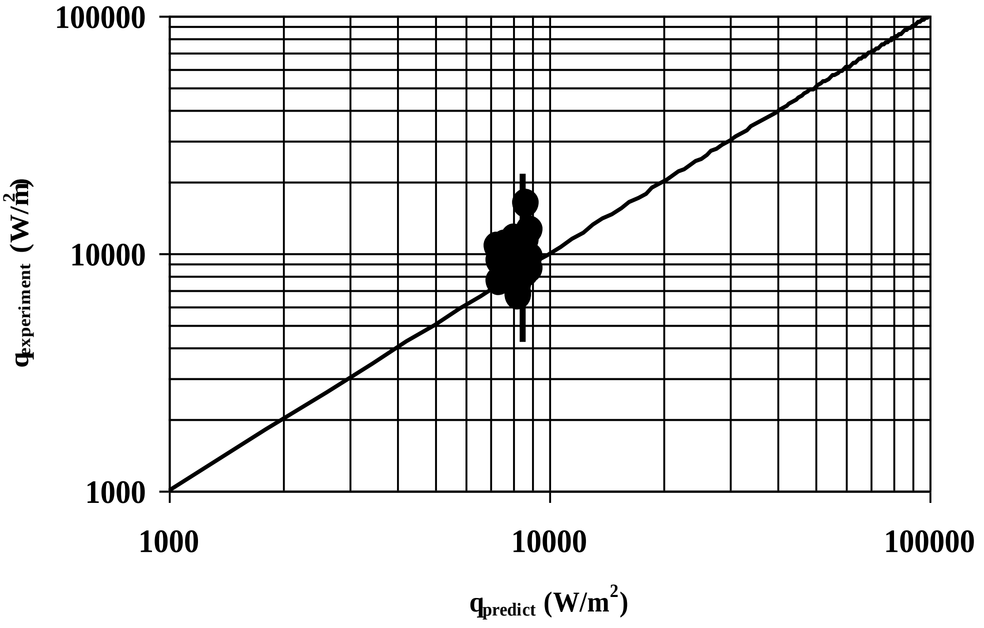

Fig. 5. Comparison of heat-transfer of prediction and that of experiment.

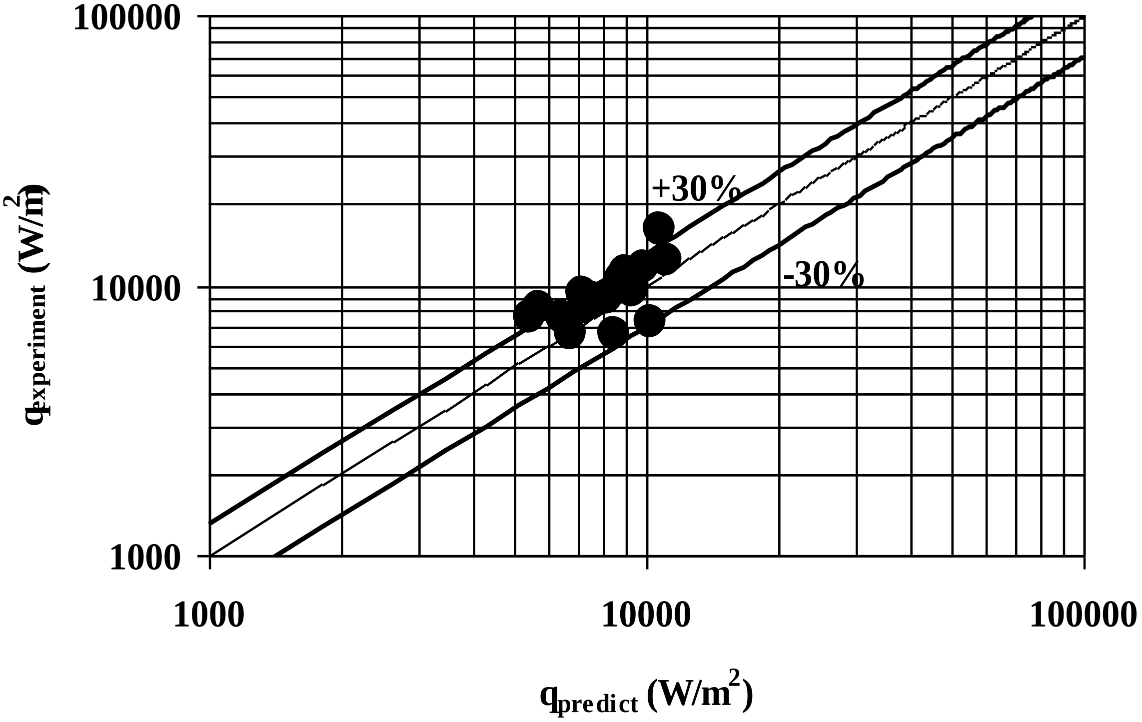

Fig. 6. Comparison of heat-transfer from the prediction and experiment.

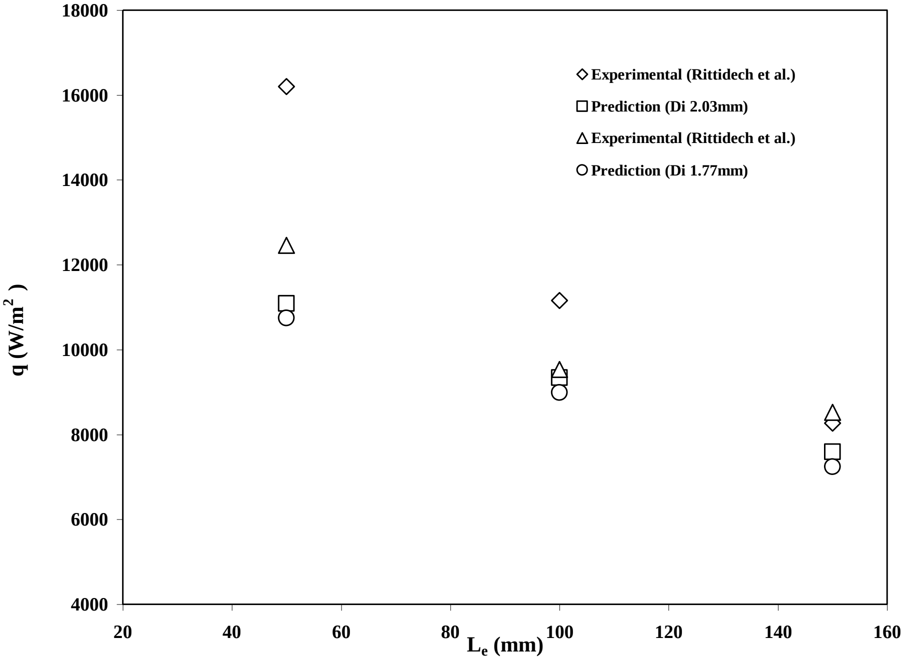

Fig. 7. Effect of evaporator section length and inner diameter on heat-transfer rate.

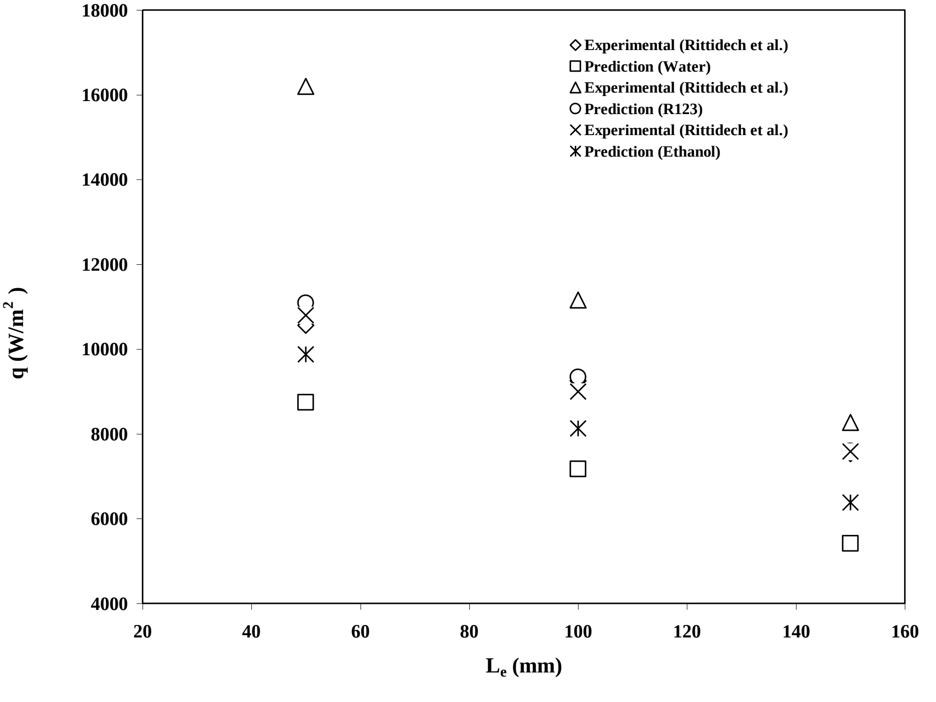

Fig. 8. Effect of evaporator section length and working fluid on heat-transfer rate.

Table 1. The results of numerical simulation and experiment from Rittidech et al. (2006)

Working fluid	D_i (mm)	L_e (mm)	$q_{pred.}$ (W/m^2)	$q_{exp.}$(W/m^2)
Water	2.03	50	8744	10560
		100	7166	9250
		150	5416	7530
R123	1.77	50	12460	10742
		100	9530	8992
		150	8520	7242
	2.03	50	16200	11088
		100	11160	9338
		150	8270	7588
Ethanol	2.03	50	9878	10800
		100	8128	9000
		150	6378	7580