

Development of Mathematical Concepts about Statistics using the Concept Formation Model together with Concept-Rich Instruction and Higher-Order Questions for Undergraduate Students Majoring in Mathematics

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Abstract: The purposes of this research are as follows: (1) to compare and study mathematical concepts about statistics among undergraduate students majoring in mathematics before and after learning by using the concept formation model together with concept-rich instruction and higher-order questions; (2) to study mathematical misconceptions about statistics on undergraduate students majoring in mathematics; and (3) to study the opinions of undergraduate students majoring in mathematics about the use of the concept formation model together with concept-rich instruction and higher-order questions. The research samples were 35 undergraduate students in their second year of college and majoring in mathematics in the Faculty of Education of Burapha University. The research instruments were composed of six lesson plans using the concept formation model with concept-rich instruction and higher-order questions, a test of mathematical concepts about statistics, and the pre-learning and post-learning versions with a reliability of 0.78 and 0.855 respectively, and the record form of mathematical misconceptions about statistics and the questionnaire about opinions towards the use of concept formation model together with concept-rich instruction and higher-order questions. The overall research findings revealed the following: (1) the mathematical concepts about statistics of undergraduate students majoring in mathematics after learning by using the concept formation model together with concept-rich instruction and higher-order questions higher than before learning with a statistical significance level of .05; (2) the mathematical misconceptions about statistics of mathematics major undergraduates were three aspects as follows: problem interpretation, the use of theories, formulas, laws, definitions, properties and computations and (3) the opinions of undergraduate students majoring in mathematics about the use of the concept formation model together with concept-rich instruction and higher-order questions was in highest level ($M=4.57$, $SD=0.51$).

Keywords: Concept formation model, Concept-rich instruction, Higher-order questions, Mathematical concepts, Mathematical misconceptions, Statistics

Introduction

Mathematics plays a vital role in the development of human thought, enabling people to have creative, logical, systematic, pattern-based thinking for being able to analyze problems or situations carefully and thoroughly, and helping people to estimate, plan, make decisions, solve problems, and apply appropriately in daily life (Angganapattarakajorn, 2012: 1). Therefore, the mathematics curriculum improvement (Revised version 2017) is in accordance with the basic education core curriculum B.E. 2551 (2008) of the Ministry of Education determined that mathematics should be contained in one of eight learning strands. The statistics and probability content strand of the strands of mathematics and statistics is an important part of a modern society. Students should have statistical skills and must have knowledge about variance, data distribution and how to interpret social situations in a general fashion to be used for decision making in different aspects (McGillivray & Pereira-Mendoza, 2011).

No matter how important mathematics and statistics are, previous national and international education quality assessments found that student knowledge of mathematics

and statistics were at a level where urgent improvement is required. According to the results of the ordinary national educational test (O-NET) in the statistical content strand, it was found that the average O-NET scores in mathematics, statistics and probability content strand in 2020 and 2021 of students finished Mattayomsuksa Six (Grade 12 students) were 25.44% and 23.09%, respectively (Bureau of Educational Testing, 2022: online). In addition, the Third International Mathematics and Science Study (TIMSS) international assessment found that in 2007, 2011 and 2015 average scores for Thai students for mathematics were 441, 427 and 431, respectively, lower than the determined international average score, 500. Consideration of the TIMSS assessment result in 2015 indicated that Thai students scored lowest on information and opportunity content, compared with other contents like numbers, algebra, and geometry (IPST, 2016: online).

Based on the previously mentioned information, Thai students needed improvement on mathematical knowledge about statistics. It is possible that students did not have knowledge and understanding what they are learning or did not understand the meaning or mathematical concepts. In this regard, they are unable to use the existing knowledge to analyze and solve problems (Makanong, 2002: 1). The Institute for the Promotion of Teaching Science and Technology (IPST) (2012: 8-9) specified that teachers should teach students to be developed in mathematical concepts to ensure students achieve meaningful learning of mathematics and be able to connect their knowledge with methods until they can apply it broadly. Additionally, the way students did not have correct understanding about mathematical concepts shall enable students to have misconceptions. Misconceptions are important problems for student learning. Li (2003: 6) stated that misconceptions are barriers to learning new content and reviewing existing knowledge.

Based on the problems related to mathematical concepts about statistics content and causes of problems mentioned above, the researcher studied teaching models that promote the development of mathematical concepts and found that concept formation model is an interesting teaching model that can be used to develop mathematical concepts, as a model that encourages students to observe, distinguish similarities and differences, interpret various concepts, and using higher-order thinking in a systematic way acting on data for structural management so as to conclude as the meaning of concepts. Meanwhile, students are able to bring understanding of concepts to use correctly (Ausubel, 1968, Lasley, Matczynski & Rowley, 2002: 177). In addition to teaching students about mathematical concepts, and another important thing is that students should be corrected misconceptions or mathematical errors that may arise in learning. Concept-rich instruction is a teaching strategy that promotes concept formation to be completed using a reflective thinking process, discussion, knowledge exchange through spoken and written language. Moreover, an examination and correction of misconceptions or errors that may arise are available (Meir Ben-Hur, 2006: 11-41). Furthermore, questions from teachers should be higher-order questions as higher-order questions shall promote higher-order thinking among students, making sure students have deep mathematical understanding, promoting students to compare, analyze, search for patterns or manage information to find out a rational conclusion or answer, and promoting students to learn new things based on their existing knowledge and experiences, and connecting the knowledge obtained from the real world (Thomas & Josepha, 1998; Center for Teaching Excellence, 2006). From the reasons mentioned above, the researcher was interested in developing mathematical concepts about statistics of undergraduate students majoring in statistics who are going to become teachers of mathematics by using a concept formation model together with concept-rich instruction

and higher-order questions. The results obtained from the research can be used as a guideline for teaching and learning management for the development of mathematical concepts based on correct statistics content, and information for making a plan to correct misconceptions in statistics that may arise, and a source of information for future research accordingly.

The research objectives of this research were as follows: (1) to compare and study mathematical concepts about statistics of undergraduate students majoring in mathematics before and after learning by using the concept formation model, together with concept-rich instruction and higher-order questions; (2) to study mathematical misconceptions about statistics of undergraduate students majoring in mathematics before, between and after learning by using the concept formation model together with concept-rich instruction and higher-order questions; and (3) to study opinions of undergraduate students majoring in mathematics about the use of the concept formation model together with concept-rich instruction and higher-order questions. The research conceptual framework, as follows:

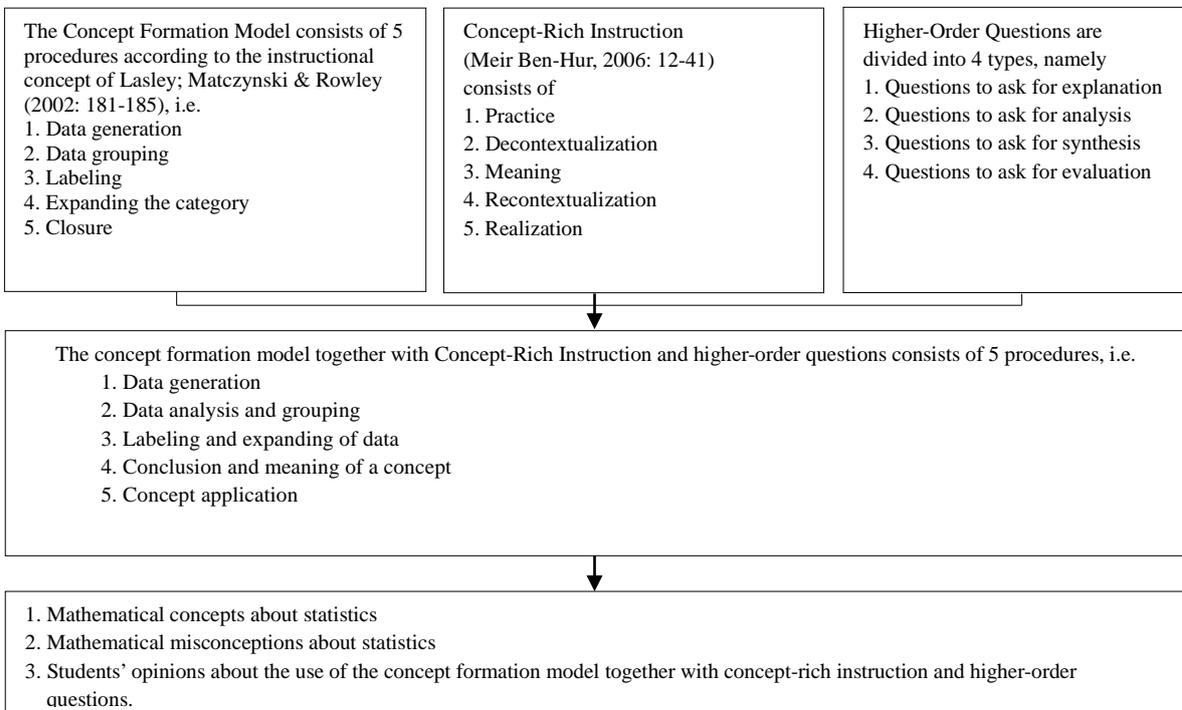


Figure 1. Conceptual framework

Research methodology

Population and sample in the research

The population in the research was undergraduate students majoring in mathematics in the Faculty of Education at Burapha University in the first semester of the 2023 academic year. The sample in the research consisted of 35 undergraduate students in their second year of college and majoring in mathematics in the Faculty of Education at

Burapha University who studied the mathematics course (40620162) for high school education in the first semester of the 2023 academic year.

Procedures to conduct the research

The research was conducted based on the five procedures, as follows:

Procedure 1: Study and analysis of basic data. This procedure involves the study and analysis of the data related to learning standards and indicators for mathematics strands (Revised version 2017) in accordance with the Basic Education Core Curriculum B.E. 2551 (2008) to analyze mathematic concepts about statistics. The study was conducted on the problems obtained from research relevant to mathematical concepts and mathematical misconceptions about statistics and problem-solving guidelines from documents, textbooks, and articles, related to concept formation model, concept-rich instruction and higher-order questions.

Procedure 2: Synthesis of basic data. This procedure involved basic data synthesis to develop the concept of the formation model, together with concept-rich instruction and higher-order questions, consisting of five steps:

Step 1: Data generation – Data related to concept formation were produced, collected and considered. Data may be obtained from data prepared by teachers or students to allow students to classify concepts.

Step 2: Data analysis and grouping – students practiced data consideration. They learned how to make a plan, manage a system, analyze data relationship, classify data having similar concepts, and describe the principles used to classify each group of data through the use of higher-order questions. Teachers should give students time to think, examine, and correct when students have misunderstandings.

Step 3: Labeling and expanding of data – Students identified a name of a group of data. The name must explain the characteristics according to the concept while connecting and covering all data in that group. The higher-order questions were used and teachers expanded student understanding of the group of data leading to the concept.

Step 4: Conclusion and meaning of a concept – It is a step that allows students to have reflective thinking and present the meaning of the concept using their own mathematical language and symbols through speaking or writing. Teachers examined the correctness of the concepts presented by students, improved the concepts to be accurate, and corrected their misunderstandings.

Step 5: Concept application – It is the step that teachers promote students to have complete and explicit concepts. Students were allowed to use the concepts to connect with their old and new experiences. Students practiced to use the concepts with the context different from the usual one. Teachers assisted, checked, and corrected students' misconceptions or misunderstandings.

Procedure 3: The research instruments consisted of:

1. The six lesson plans used the concept formation model together with concept-rich instruction and higher-order questions. The evaluation result of the congruence between components of all six lesson plans (IOC), measured by experts in teaching mathematics, was 1.00. It can be concluded that all learning management plans were consistent with various components.

2. There are two versions of the test of mathematical concepts about statistics, pre-learning version and post-learning version. The test came in the form of 25 subjective test items. The two versions of the test had parallel questions in each item. The evaluation result of the congruence between each question item and the mathematical concepts measured (IOC) of the pre-learning version and post-learning versions, measured by the experts in teaching mathematics was 1.00, meaning that all question items in the pre-learning version and in the post-learning version were consistent with the mathematical concepts measured. The reliability of the pre-learning version and the post-learning version was 0.78 and 0.855, respectively.

3. The record form of mathematical misconceptions about statistics. It was used to record misconceptions found in the learning management and the checking of the pre-learning version and post-learning version measurement test. The experts in teaching mathematics viewed that the record form is accurate and appropriate and can be used to record mathematical misconceptions about statistics.

4. The questionnaire about opinions towards the use of concept formation model together with concept-rich instruction and higher-order questions. It is a five-rating scale questionnaire with 20 question items, which the experts in teaching mathematics viewed that it is accurate and appropriate.

Procedure 4: Data collection.

Time spent on data collection in this study was 24 hours, divided into three hours of the pre-test, 18 hours of teaching according to the six lesson plans, and three hours of the post-test. The implementation is as follow:

1. Before teaching, students were assigned to take the test of mathematical concepts about statistics (pre-learning version). The researcher checked the answers of each student, assigned a score, and recorded mathematical misconceptions about statistics in the record form. The scores on mathematical concepts about the statistics for each question were categorized into four groups, as follows:

Table 1. Mathematical concepts score system

Group 4	Correct answers with an explanation referred to mathematical knowledge and mathematical concept relationships that can explicitly support answers.	3 points
Group 3	- Correct answers with the explanations referring to mathematical knowledge and mathematical concept relationships that can support the answers but not explicit. - Answers that are partially correct, but with explicit explanation referring to mathematical knowledge and mathematical concept relationships.	2 points
Group 2	- Correct answers, but without any explanation or attempt to give some explanation referred to mathematical knowledge and mathematical concept relationship to support the answers, but not correct. - Answers that are partially correct or incorrect answers, but with some explanation referred to mathematical knowledge and mathematical concept relationship to support the answers but not explicit.	1 point
Group 1	Answers that are partially correct or incorrect answers or no answer without any explanation or attempt to give some explanations referred to mathematical knowledge and mathematical concept relationships to support the answers, but not correct.	0 point

2. Students who were the research sample were taught by the researcher. The teaching took 18 hours. While the teaching was performed, the researcher recorded mathematical misconceptions about statistics in record forms.

3. After the teaching ended, the students were assigned to take a test of mathematical concepts about statistics (post-learning version). The researcher checked answers of each student using the same scoring criteria as seen in the pre-learning version, and recorded mathematical misconceptions about statistics in the record form.

4. The students were assigned to respond to the questionnaire about opinions towards the use of concept formation model together with concept-rich instruction and higher-order questions. The answers were checked to evaluate student opinions.

Procedure 5: Both quantitative and qualitative data analysis were performed in this study, as follows:

1. The comparison and study of mathematical concepts about statistics. Quantitative data, i.e. student scores obtained from the measurement test of mathematical concepts about statistics, before and after learning, were analyzed using percentage and a dependent t-test.

2. The study of mathematical misconceptions about statistics and the qualitative data obtained from the record forms were analyzed to find out mathematical misconceptions about statistics.

3. The study of students' opinions towards teaching and learning. The quantitative data, i.e. the scores obtained from the questionnaire and analyzed using mean (M) and standard deviation (SD).

Research results

Part 1. The comparison and study of mathematical concepts about statistics of undergraduate students majoring in mathematics.

The comparison and study results of mathematical concepts about the statistics of undergraduate students majoring in mathematics, before and after learning, using the concept formation model together with concept-rich instruction and higher-order questions are shown in Table 2-3.

Table 2. The comparison of mathematical concepts about statistics of students before and after learning using the model.

Mathematical concepts about statistics	The number of students	Mean score (M) (full score = 75 points)	Standard deviation (SD)	t	p-value
Before Learning	35	28.49	8.74	19.619*	.00
After Learning	35	59.00	4.18		

* $p < .05$

From Table 2, it was found that undergraduate students majoring in mathematics had a higher level of mathematical concepts about statistics after learning, using the

concept formation model together with concept-rich instruction and higher-order questions than before learning with a statistical significance level of .05.

Table 3. Percentage of the students before and after learning using the model, classified by the groups of scoring criteria for mathematical concepts about statistics.

Mathematical concepts about statistics	Percentage of the students in Group 1	Percentage of the students in Group 2	Percentage of the students in Group 3	Percentage of the students in Group 4	Total
Before Learning	28.80	34.97	29.72	6.51	100
After Learning	0.68	14.29	33.37	51.66	100

From Table 3, it was found that before learning, undergraduate students majoring in mathematics had mathematical concepts about statistics in Group 2 at the highest (34.97%) while after learning undergraduate students majoring in mathematics had mathematical concepts about statistics in Group 4 at the highest (51.66%).

Part 2. The study of mathematical misconceptions about statistics of undergraduate students majoring in mathematics.

The study results of mathematical misconceptions about statistics of undergraduate students majoring in mathematics indicated that the students had mathematical misconceptions in all 3 aspects studied. Details are as follow:

1. Problem interpretation – students had mathematical misconceptions about not using the information determined by math questions. They misused the information determined by the math questions, they did not consider what the questions determined and incorrect information and the questions did not determine to find an answer.

2. The use of theories, formulas, laws, definitions and properties. Students had mathematical misconceptions about the definition of descriptive statistics, the definition of inferential statistics, statistical questions, the definition of qualitative data, the definition of quantitative data, a class interval in a frequency distribution table, data presentation using a frequency distribution table, the distribution of data presented in a box plot, using dot plots to display the distribution of data, data presentation in tables, using stem-leaf plots for data presentation, presenting data in different forms, the relationships between quartiles and percentiles, the position of a decile, the definition of a percentile, the definition of a decile, the position of a percentile, the relationship between decile and percentile, how to find the position of a quartile of non-frequency data, quartile value and quartile position, how to calculate the midpoint of a class interval which have the lowest or the highest of the class interval as an open class interval, how to find the median of data with distribution of frequency, how to find the central tendency of qualitative data, the meaning of measuring the central tendency of data, formulas to find the median of data with distribution of frequency, how to find the central tendency of quantitative data, how to find the central tendency of data using arithmetical mean, how to find the central tendency of data using the mode, how to find the median position of data with frequency distribution, the central tendency of data, how to find the central tendency of data with frequency distribution, the meaning of standard deviation equal to zero, the arithmetic mean and standard deviation of non-frequency data that increase, quartile deviation, data quantity and the measures of data

distribution, standard deviation formula, the meaning of measures of relative variation, normal distribution, normal curve, the variance of data, right-skew curve and left-skew curve.

3. Computations. Students had mathematical misconceptions about basic mathematical principles such as $\frac{53}{7} = 7.6$ or $\frac{3}{4}(20 + 1) = 16$ or $5 \left[\frac{10-4}{11} \right] = 2.77$, or $\sqrt{2.5} = 0.5$ or $0.5 \times 0.8 = 4$ and were deprived of understanding of algebra such as from

$$0.4 = 32 - x \text{ will get. } x = \frac{32}{0.4} = 80$$

Part 3. The study of opinions of undergraduate students majoring in mathematics towards concept formation model together with concept-rich instruction and higher-order questions.

Overall, undergraduate students majoring in mathematics had the highest level of opinions towards the concept formation model together with concept-rich instruction and higher-order questions (M=4.57, SD=0.51). Consideration of each aspect found the students viewed that the teaching process promoted correct and explicit mathematical concepts about statistical questions, data, data collection, and frequency distribution of data. Furthermore, the teaching and learning process promoted correct and explicit mathematical concepts about data presentation and data analysis at the maximum level (M=4.80, SD=0.41). Next, the students viewed that the teaching process promoted correct and explicit mathematical concepts about how to measure the central tendency of data (M=4.77, SD=0.43), and the students viewed that knowledge discussion and doing activities with groups of friends enabled them to have increased understanding of mathematical concepts about statistics at the minimum level (M=4.14, SD=0.55).

Discussion

The findings from this research were obtained from the way the researcher developed mathematical concepts about statistics of 35 undergraduate students in their second year of college majoring in mathematics, Faculty of Education, Burapha University, in the first semester of the 2023 academic year, based on the concept formation model together with concept-rich instruction and higher-order questions. Research results discussion is shown as follow:

1. The undergraduate students majoring in mathematics had higher mathematical concepts about statistics after learning with the concept formation model together with concept-rich instruction and higher-order questions than before learning with a statistical significance level of .05. This is consistent with a study result of Wimonrat Srisuk (2008) indicating that students' mathematical knowledge after learning with the instructional process integrating the concept formation model and the translation model was higher than before learning with a statistical significance level of .05. A research result from Deogratias, (2020: ii-iii) found that concept-rich instruction helped develop understanding of mathematical concepts of student teachers in mathematics. It was possible that the

concept formation model together with concept-rich instruction and higher-order questions developed by the researcher, consisting of 5 steps, could help promote the development of mathematical concepts about statistics as follow:

Step 1: Data production – This is the beginning step important to the development of students' conception since data preparation is considered teachers' explicit goal determination what they would like students to achieve learning in which concepts. The prepared data must be large enough to enable students to see co-dominance of concepts to be made. This process promotes the development of mathematical concepts as what Laphatpinyochok, (2015: 88-89) said that a guideline to develop mathematical concepts was determining learning goals that expected to occur to students for which goal they are going to achieve. What kind of concepts students need to know to achieve the goals and teachers have to prepare data or situations consistent with the characteristics of a certain concept for students to study, discover and learn by themselves.

Step 2: Data analysis and classification – the activities implemented in this step is important in the development of the conceptions of students, since they need to determine a criterion to classify data that share the same characteristics, which lead to a summary of concepts by themselves. This is consistent with what Mekanong (2014: 23) said that the way students did activities, thought, observed, analyzed, discussed, and found a conclusion in mathematics by themselves was a method to develop mathematical concepts. White & Mitchelmore (2002: 2-6) said that observing the similarity of a relationship process or objects related to concepts would lead to the concept formation and conclusions. Besides, the use of higher-order questions could help promote students to use critical thinking in logic, reasoning, and creativity for data classification since higher-order questions could promote higher-order thinking and making students understand mathematics more deeply (Schmalz, 1978).

Step 3: Data identification and explanation of types of data – It is the step that allows students to identify name of groups of data that explain the characteristics in accordance with concepts, covering all data in a certain group. Activities in this stage are able to develop concepts as what Asubel (1968: 517) said that making a conclusion being able to cover specific characteristics of new concepts and all small parts of a group was a part of concept formation and development. Additionally, the way teachers expanded understanding by giving more information and students analyzed and gave reasons in which group the data should be classified would encourage students to think and consider again what kind of important co-dominance that the group of data following a certain concept had and if the examples given by the teachers shared any similar characteristics to encourage students to crystalize their own idea and gain clarity in the characteristics that those concepts shared.

Step 4: Conclusion and meaning of a concept – It is an important step for promoting understanding, and correct mathematical concepts. Many educators said that the process of reflective thinking, knowledge exchange, presentation, conclusion or definition of concepts through spoken and written language and examples of concepts were an important guideline for mathematical concept development (Meir Ben-Hur, 2006: 11-41; De Cecco, 1968: 402-416; Laphatpinyochok, 2015: 88-89). Additionally, the way teachers

checked the correctness of concepts presented by students and improved them is an evaluation to consider whether the concepts concluded by students are correct or not and if they have any misconceptions, teachers are able to correct them immediately.

Step 5: Concept application – It is a step that teachers use to promote students to have complete and explicit concepts by allowing them to use the concepts to connect previous experience with new experience. Students had a chance to practice using concepts with different context or in real-life situations. This is consistent with a guideline for mathematical concept development introduced by Klausmeier & Ripple (1971: 422-432) showing that teachers played a part in helping students to apply the concepts they learned in a useful way including using the concept for solving problems and had a chance for decision making.

2. According to the study results, the undergraduate students majoring in mathematics had mathematical misconceptions in three aspects studied, namely, problem interpretation, using theories, formulas, laws, definitions, properties and computations. This is consistent with study results of Angganapattarakajorn (2020: 88) on the characteristics of mathematical misconceptions about statistics of undergraduate students majoring in mathematics, Faculty of Education, Burapha University as it was found that the students had mathematical misconceptions about statistics in three aspects, i.e. problem interpretation, the use of theories, formulas, laws, definitions, properties and computations. However, after learning with the concept formation model together with concept-rich instruction and higher-order questions, students decreased mathematical misconceptions, probably due to the concept formation model together with concept-rich instruction and higher-order questions developed by the researcher. In step 2, students practiced skills and considered all data. They made a plan, manage a system, analyzed data relationship and classified data having similar concept characteristics. Teachers gave students time to think, check and correct together when they had misconceptions. In step 4, students had reflective thinking and presented the definition of concepts using their own mathematical language and symbols. Next, teachers checked accuracy of the concepts presented by students, improved the concepts, and cleared up misunderstanding. In step 5, teachers promoted students to have complete and explicit concepts by allowing students to practice using the concepts with different context or in real- life situations. Teachers helped students by checking and correcting misconceptions or misunderstanding.

3. According to the study results, it could be seen that undergraduate students majoring in mathematics had the highest level of opinions towards the concept formation model together with concept-rich instruction and higher-order questions ($M=4.57$, $SD=0.51$). This is consistent with the study results of Makanong (2009: 96) showing that opinions of students majoring in mathematics and primary education with mathematics towards the teaching and learning process using the concept attainment model and higher-order questions were at a high level ($M=4.85$, $SD=0.33$). It was possible that learning activity management using the concept formation model together with concept-rich instruction and higher-order questions is learning management that focuses on knowledge creation and mathematical concepts about statistics based on student understanding. The way that students created their own knowledge made students have better attitudes towards

mathematics. In this regard, they had positive opinions towards mathematics subjects and mathematics learning activities developed by the researcher.

Conclusion

The concept formation model, together with concept-rich instruction and higher-order questions, is a teaching model that promotes development of mathematical concepts about the statistics of undergraduate students majoring in mathematics, who were the research sample, and helps adjust the misconceptions of students. The opinions of students towards this teaching model were at the highest level. Therefore, mathematics teachers are able to employ this concept formation model together with concept-rich instruction and higher-order questions for teaching and learning management giving students opportunities to practice, make a plan, analyze, reflect on their thinking, including creating and summarizing knowledge and mathematical concepts. Teachers should use higher-order questions to stimulate and promote students to think, manage a system and seek relationships to make a conclusion of mathematical concepts correctly.

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