Time Series Analysis of Demographic Parameters in Bangladesh

Md. Sabbir Hossain[1]*, Md. Rafiqul Islam[2], and M. Korban Ali[3]

[1] Department of Business Administration, Bangladesh Islami University, Dhaka, Bangladesh
[2] Department of Population Science and Human Resource Development, University of Rajshahi, Rajshahi, Bangladesh
[3] Department of Statistics, University of Rajshahi, Rajshahi, Bangladesh
* Md. Sabbir Hossain, corresponding author. Email: drmdsabbirhossain@gmail.com
Submitted: 20 November 2023. Accepted: 4 December 2024. Published: 20 March 2025
Volume 34, 2026. pp. 134–157. http://doi.org/10.25133/JPSSv342026.008

Abstract

Demographic parameters focus on the overall health status of a country. These are necessary indicators for analyzing the health status of the sustained population in Bangladesh. Econometric models fitted on six important demographic parameters separately, three of six parameters for mortality measures and the remaining for fertility measures of Bangladesh. Autoregressive models developed on the time series data of demographic parameters such as life expectancies at birth for male and female populations, crude death rates, crude birth rates, gross reproduction rates, and net reproduction rates by year from 1980 to 2015 collected from Statistical Yearbook of Bangladesh published by the Bangladesh Bureau of Statistics. Mortality and fertility measures were predicted up to 2030 using the fitted models. The crude death rates, crude birth rates, gross reproduction rates, and net reproduction rates decreased by years from 2016 to 2030, while life expectancies at birth for the male and female populations were increasing. Government and non-government organizations and policymakers can make several decisions for more development of the sectors such as health, education, planning of food supply, and housing. Government and insurance companies in Bangladesh can also utilize the results of life expectancies at birth for the male and female population in setting the retirement age of government employees, fixing the minimum age of old age allowance, and innovating age-related rules of life insurance companies.

Keywords

Autoregressive model; demographic parameters; econometric model; fertility; mortality; time series analysis

Introduction

Time series analysis of demographic parameters at different time intervals is a common practice in social sciences and economics to estimate or predict future trends or to understand the changes in variables over time. Shubat and Bagirova (2022) studied the length of grandparenthood using methods of mathematical modeling and trend extrapolation based on time series analysis and forecasted the length of grandparenthood in Russia. Robinson and Jensen (2020) used different methods to observe trends and outliers in time series data. The United Nations Development Programme (1991) highlighted in the report that high life expectancy is the key indicator of socio-economically developed countries.

Denton et al. (2005) studied time series data of mortality and life expectancies using Canadian data, fitted models of mortality and forecasted life expectancies, and estimated age-sex group system models. Quantitative implications of forecasted life expectancies are investigated, and experimental stochastic forecasts are discussed based on the Lee-Carter method. Yang et al. (2022) studied time series data from 2000 to 2018 and observed the dynamic relationship between life expectancy and its influencing factors in Beijing City, China. A Vector Autoregression model was developed, and results showed that a Vector Autoregression model with optimum lag 3 was constructed between life expectancy and three explanatory variables.

Halicioglu (2011) studied time series data of life expectancy in Turkey for the period 1965–2005 to understand the factors that influence life expectancy. The Bounds-testing approach to cointegration is employed to compute the long-run elasticities of longevity concerning the selected economic, social, and environmental factors. Salvati et al. (2020) studied fertility measures such as total fertility rate and crude birth rate at different geographical scales between 2002 and 2018 in Italy. They investigated the spatiotemporal evolution of these parameters. Results showed that fertility decreased following an opposite trend concerning economic conditions. Fertility is the key to the demographic process that contributes to population trends (Balbo et al., 2013; Goldstein et al., 2009).

The American Community Survey also utilized time series data to observe changes in the characteristics of the population over time (Molugaram & Rao, 2017). The Autoregressive time trend model is fitted for predicting the population growth of Bangladesh (Beg & Islam, 2016). Time series analysis was conducted on the crude death rates of Bangladesh in which econometric models, Augmented Dickey-Fuller, Autoregressive Distributed Lag bounds, and pairwise Granger causality tests were used, and indicated a relationship among crude death rates and demographic parameters, and also described the necessity of mortality measures for insurance companies in Bangladesh (Begum, 1990; Mannan et al., 2014).

Muyeed et al. (2020) investigated the prevalence and changes in disease events, COVID-19, in Bangladesh using daily time series data. The events associated with COVID-19, such as the daily cases, case fatality rate, recovery-death-ratio, and percent changes, were used for prevalence and trends, and it found that 68% of male and 32% of female patients were infected. Rahman and Alam (2023) studied child mortality in Bangladesh from the data from 1975 to 2019. They applied several econometric time series analysis techniques, such as the Augmented Dickey-Fuller test, Autoregressive Distributive Lag bounds, and pairwise Granger causality tests. Among all of them, they chose the Augmented Dickey-Fuller test. Total fertility rate and urbanization have a positive effect on the child mortality rate, whereas female education, female life expectancy at birth, and economic growth rate have a negative impact.

Several studies related to life expectancies and birth rates were done, and the results of these studies showed that improvements in survival play a necessary role in explaining changes in the net reproduction rate over time in some low-income countries (Islam et al., 2005; Rubi et al., 2021; Shen et al., 2023). Nandi et al. (2023) conducted a study to investigate and measure the effect of socioeconomic variables such as employment rate, gross national income, population growth rate, unemployment rate, and age dependency ratio on life expectancy in Bangladesh using autoregressive integrated moving average models. In several studies, econometric models are used in time series data (Aksoy et al., 2019; Guets & Behera, 2022; Kelley & Schmidt, 2005; Matthews & Parker, 2013; Voss, 2007; Winkelmann & Zimmermann, 1994; Xie, 2000).

This study aims to construct econometric models for mortality measures such as crude death rate (CDR), life expectancy at birth for the male population (LEBM) and life expectancy at birth for the female population (LEBF), and fertility measures like crude birth rate (CBR), gross reproduction rate (GRR) and net reproduction rate (NRR), and to predict mortality and fertility measures of Bangladesh up to 2030. Another important aim of this study is to analyze trends of these parameters over the years. This investigation would benefit Bangladesh by allowing it to take proper steps to optimize the socioeconomic variables in the future to achieve sustainable development goals.

Data and data source

Data utilized in this study have been collected from the Statistical Yearbook of Bangladesh published by the Bangladesh Bureau of Statistics. Bangladesh Bureau of Statistics conducts a population and housing census every ten years. During the intercensal period, the Sample Vital Registration System (SVRS) is designed to determine the annual population change. Out of six variables, three are mortality measures such as CDR, LEBM, and LEBF, and the remaining variables are fertility measures such as CBR, GRR, and NRR in Bangladesh from 1980 to 2019, excluding LEBM and LEBF in 1980, 1999, 2000 and 2001, collected from Statistical Yearbooks of Bangladesh (Bangladesh Bureau of Statistics, 1989–2012, 2013–2023).

Some data need to be included and collected from reliable sources. For these missing data, LEBM and LEBF in 1999, 2000, and 2001 were estimated using Newton's Divided Difference formula, and LEBM and LEBF in 1980 were also estimated using the Expectation Maximization (EM) technique. All of the data mentioned above utilized in this study are presented in Table 1.

Table 1: CDR, LEBM, LEBF, CBR, GRR, and NRR of Bangladesh in 1980–2015

Year	CDR	LEBM	LEBF	CBR	GRR	NRR
1980	10.2	52.8	51.4	33.4	2.41	1.92
1981	11.5	55.3	54.4	34.6	2.45	1.89
1982	11.9	54.5	54.8	34.8	2.54	1.98
1983	12.3	54.2	53.6	35.0	2.45	1.92
1984	12.3	54.9	54.7	34.8	2.34	1.81
1985	12.0	55.7	54.6	34.6	2.20	1.79

Year	CDR	LEBM	LEBF	CBR	GRR	NRR
1986	11.9	55.2	55.3	34.4	2.29	1.80
1987	11.5	56.9	56.0	33.3	2.14	1.69
1988	11.3	55.9	54.4	33.2	2.21	1.74
1989	11.4	56.0	55.1	33.0	2.10	1.72
1990	11.3	56.4	55.4	32.8	2.10	1.71
1991	11.2	56.5	55.7	31.6	2.06	1.70
1992	11.0	56.8	55.9	30.8	2.03	1.62
1993	10.0	57.8	56.6	28.8	2.01	1.57
1994	9.0	58.2	57.9	27.8	1.81	1.48
1995	8.4	58.4	58.1	26.5	1.68	1.46
1996	8.1	59.1	58.6	25.6	1.66	1.46
1997	5.5	60.5	59.9	21.0	1.52	1.37
1998	4.8	60.7	60.5	19.9	1.45	1.31
1999	5.1	60.7	61.4	19.2	1.29	1.25
2000	4.9	62.2	63.6	19.0	1.27	1.24
2001	4.8	64.1	65.3	18.9	1.26	1.23
2002	5.1	64.5	65.4	20.1	1.26	1.22
2003	5.9	64.3	65.4	20.9	1.24	1.20
2004	5.8	64.4	65.7	20.8	1.21	1.18
2005	5.8	64.4	65.8	20.7	1.19	1.17
2006	5.6	65.4	67.8	20.6	1.17	1.15
2007	6.2	65.5	67.9	20.9	1.17	1.14
2008	6.0	65.6	68.0	20.5	1.11	1.09
2009	5.8	66.1	68.7	19.4	1.07	1.06
2010	5.6	66.6	68.8	19.2	1.05	1.04
2011	5.5	67.9	70.3	19.2	1.04	1.03
2012	5.3	68.2	70.7	18.9	1.05	1.04
2013	5.3	68.8	71.2	19.0	1.02	1.01
2014	5.2	69.1	71.6	18.9	1.05	1.04
2015	5.1	69.4	72.0	18.8	1.05	1.00
2016	5.1	70.3	72.9	18.7	1.02	1.00
2017	5.1	70.6	73.5	18.5	1.02	1.00
2018	5.0	70.8	73.8	18.3	1.00	0.99
2019	4.9	71.1	74.2	18.1	1.00	1.00

Methods and methodological issues

Construction of econometric models of demographic parameters

Senturk and Ali (2021) studied time series data such as socioeconomic determinants and gender-specific life expectancies from 1971 to 2017, and the stationarity of data was checked by Unit Root tests such as the Augmented Dickey-Fuller (ADF), the Phillips-Perron (PP), and the Dickey-Fuller-generalized least-squares regression (DF-GLS). An Autoregressive Distributed Lag (ARDL) bound test was also conducted to identify cointegration (Sam et al., 2019). All variables that are under study may have some disturbances. Econometric theory is necessary for assessing the suitability of variables before fitting an econometric model. The models were estimated based on observed data, and the estimated models were tested for their suitability.

Normality and outlier check

Maddala (1992) showed that an outlier was visible at an abnormal distance from other values. The outlier of an observation depends partially on the normal distribution of the data. If data is tested for normality assumption and is not valid, then this may happen due to the non-normality of the data rather than the presence of an outlier. Applying a normal probability plot of the data is recommended before an outlier test.

Data have been investigated using skewness, kurtosis z-values, and visual outputs such as histograms and normal Q-Q plots. A visual inspection of their histograms and normal Q-Q plots in Figures 1–12 showed that CDR, LEBM, LEBF, CBR, GRR, and NRR are not normally distributed. A skewness of 0.315 (SE = 0.393) and kurtosis of -1.771 (SE = 0.768) for CDR; a skewness of 0.166 (SE = 0.393) and kurtosis of -1.415 (SE = 0.768) for LEBM; a skewness of 0.219 (SE = 0.393) and kurtosis of -1.495 (SE = 0.768) for LEBF; a skewness of 0.340 (SE = 0.393) and kurtosis of -1.753 (SE = 0.768) for CBR; a skewness of 0.288 (SE = 0.393) and kurtosis of -1.532 (SE = 0.768) for GRR; a skewness of 0.257 (SE = 0.393) and kurtosis of -1.417 (SE = 0.768) for NRR where Standard Error (SE) also show that all data utilized in this study are rarely normally distributed based on the values of skewness.

These results suggest that all variables may be normally distributed after performing difference or box-cox transformation. Scatter plots and box plots are the most well-known graphical techniques for identifying outliers, along with an analytic procedure for when the distribution is normal. Describing the behavior of the data box plot is necessary, as it indicates that any point beyond an outer fence is considered an extreme outlier. Table 2 and Figures 13–18 show that no single outlier was found in the data set of Bangladesh's CDR, LEBM, LEBF, CBR, GRR, and NRR from 1980 to 2015.

Table 2: Quartile Value, Median, Interquartile Range, Lower Inner Fence, Upper Inner Fence, Lower Outer Fence, and Upper Outer Fence of CDR, LEBM, LEBF, CBR, GRR, and NRR of Bangladesh in 1980–2015

	First Quartile	Third Quartile	Median	Interquartile Range	Lower Inner Fence	Upper Inner Fence	Lower Outer Fence	Upper Outer Fence
CDR	5.35	11.3	6.1	5.95	-3.58	20.23	-12.50	29.15
LEBM	56.1	65.47	60.6	9.37	42.05	79.53	27.99	93.58
LEBF	55.32	67.87	60.2	12.55	36.50	86.70	17.67	105.52
CBR	19.25	33.15	20.95	13.9	-1.60	54.00	-22.45	74.85
GRR	1.17	2.13	1.485	0.96	-0.27	3.57	-1.71	5.01
NRR	1.1425	1.7175	1.34	0.58	0.27	2.59	-0.60	3.46

Figure 1: Histogram of CDR of Bangladesh in 1980–2015. X: CDR, and Y: Frequency

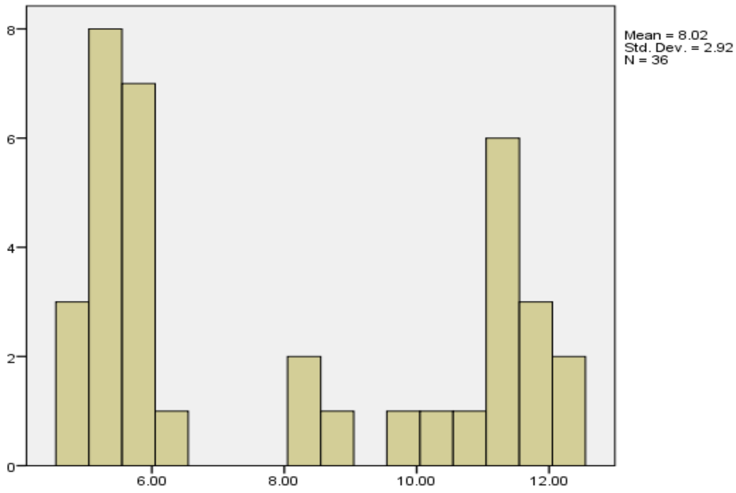


Figure 2: Normal Q-Q Plot of CDR of Bangladesh in 1980–2015. X: Observed Value, and Y: Expected Normal

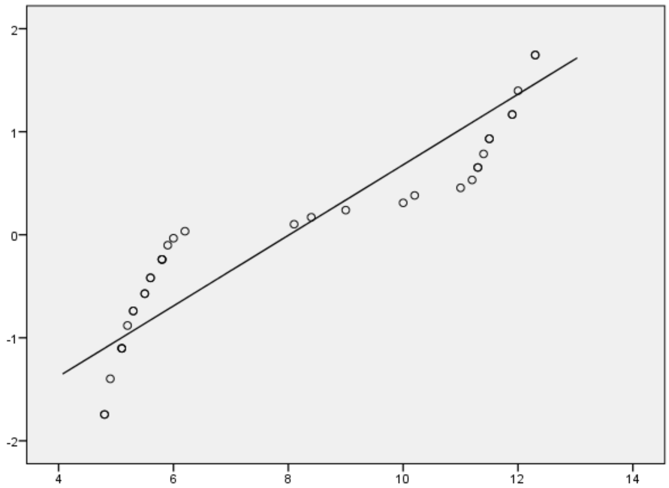


Figure 3: Histogram of the LEBM Population of Bangladesh in 1980–2015. X: LEBM, and Y: Frequency

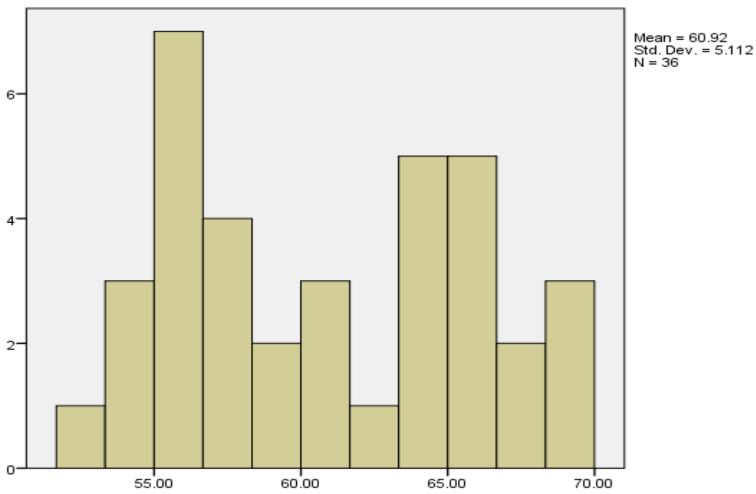


Figure 4: Normal Q-Q Plot of LEBM Population of Bangladesh in 1980–2015. X: Observed Value, and Y: Expected Normal

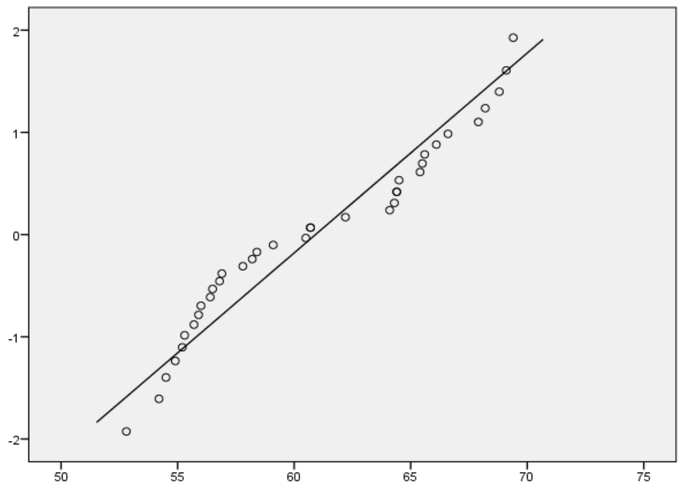


Figure 5: Histogram of LEBF Population of Bangladesh in 1980–2015. X: LEBF, and Y: Frequency

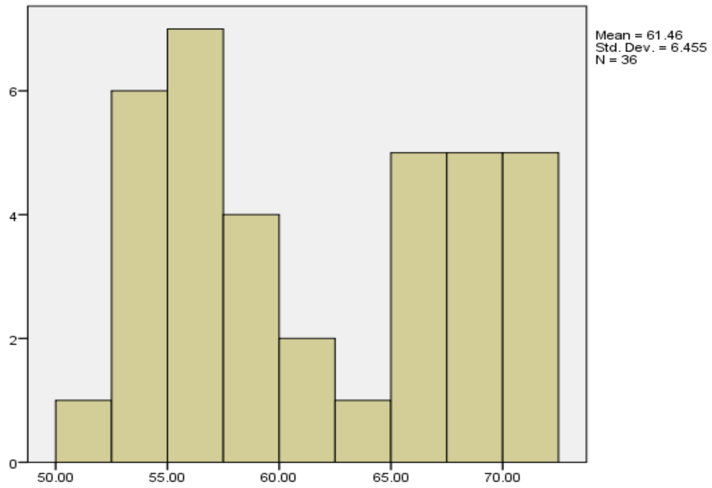


Figure 6: Normal Q-Q Plot of LEBF Population of Bangladesh in 1980–2015. X: Observed Value, and Y: Expected Normal

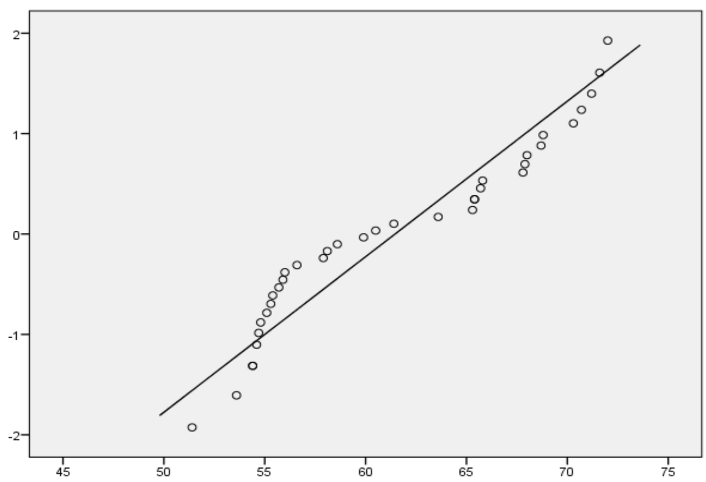


Figure 7: Histogram of CBR of Bangladesh in 1980–2015. X: CBR, and Y: Frequency

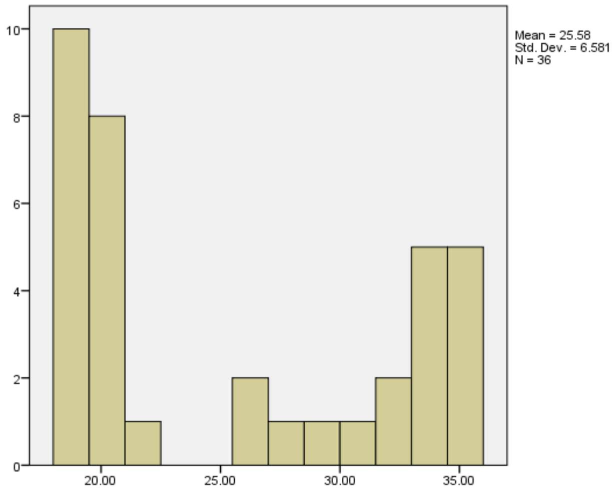


Figure 8: Normal Q-Q Plot of CBR of Bangladesh in 1980–2015. X: Observed Value, and Y: Expected Normal

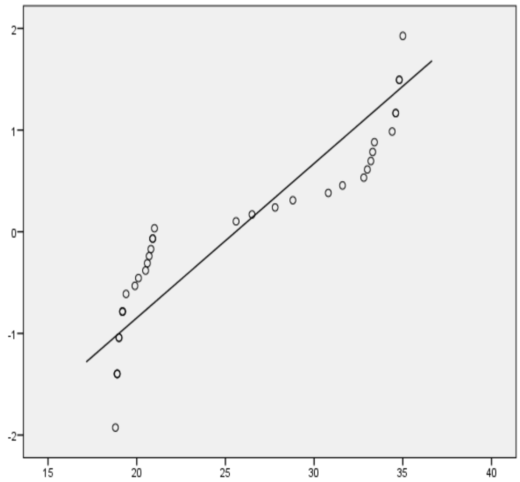


Figure 9: Histogram of GRR of Bangladesh in 1980–2015. X: GRR, and Y: Frequency

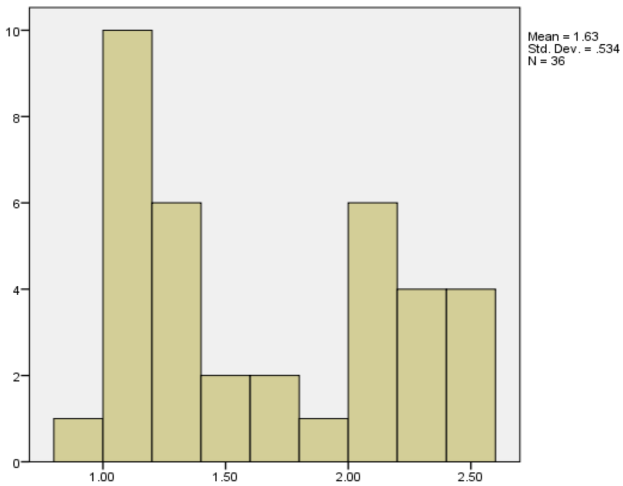


Figure 10: Normal Q-Q Plot of GRR of Bangladesh in 1980–2015. X: Observed Value, and Y: Expected Normal

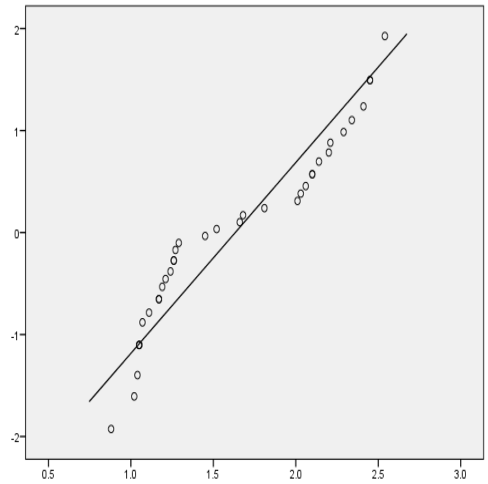


Figure 11: Histogram of NRR of Bangladesh in 1980–2015. X: NRR, and Y: Frequency

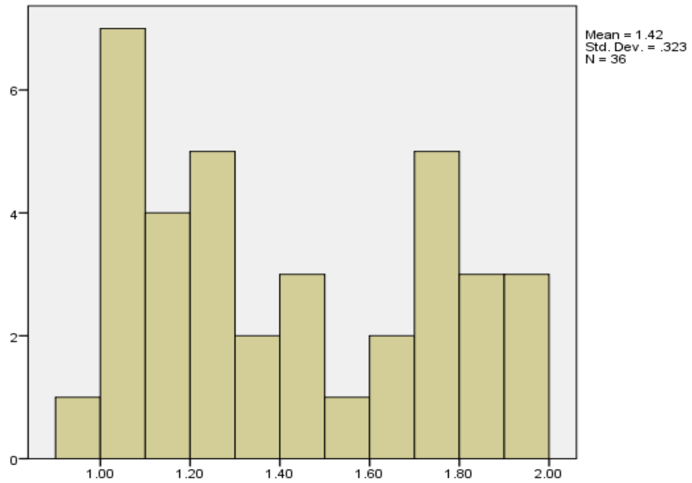


Figure 12: Normal Q-Q Plot of NRR of Bangladesh in 1980–2015. X: Observed Value, and Y: Expected Normal

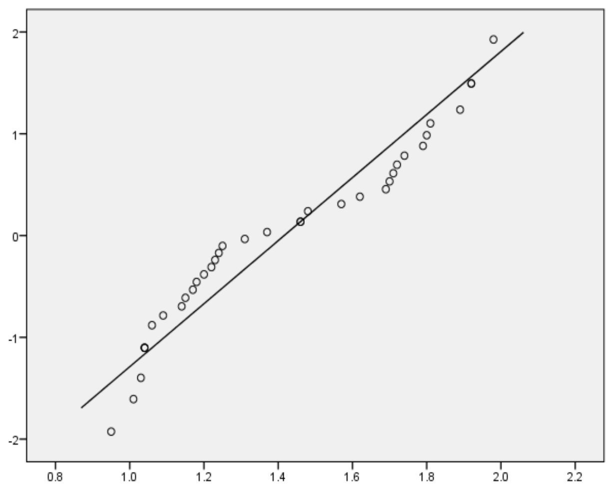


Figure 13: Boxplot of CDR of Bangladesh in 1980–2015

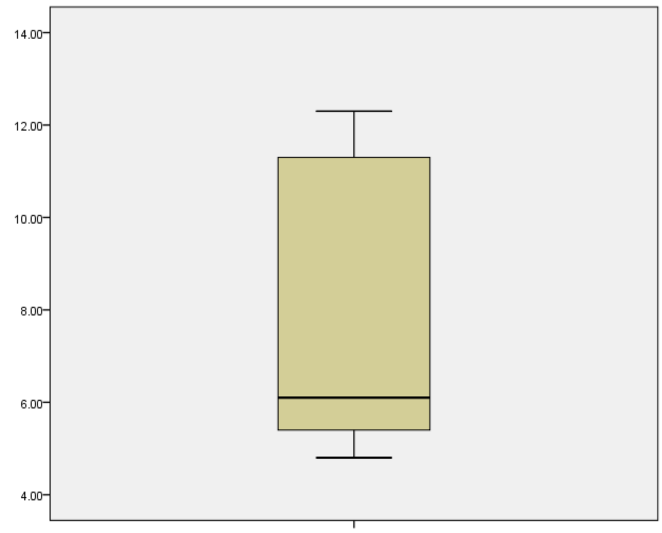


Figure 14: Boxplot of LEBM Population of Bangladesh in 1980–2015

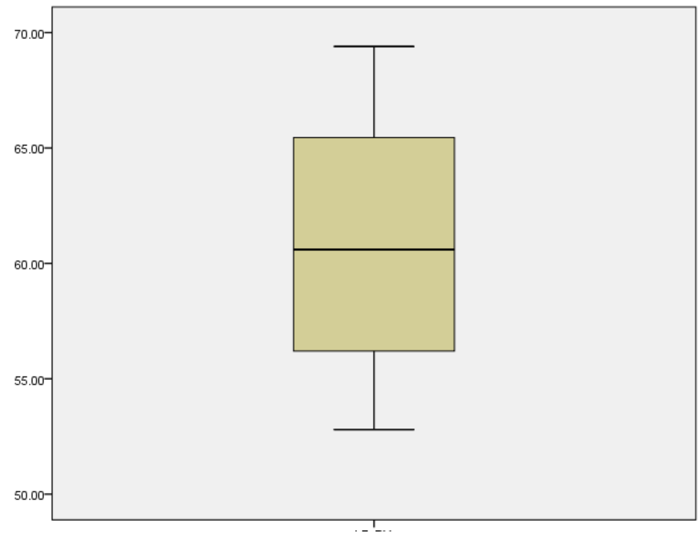


Figure 15: Boxplot of LEBF Population of Bangladesh in 1980–2015

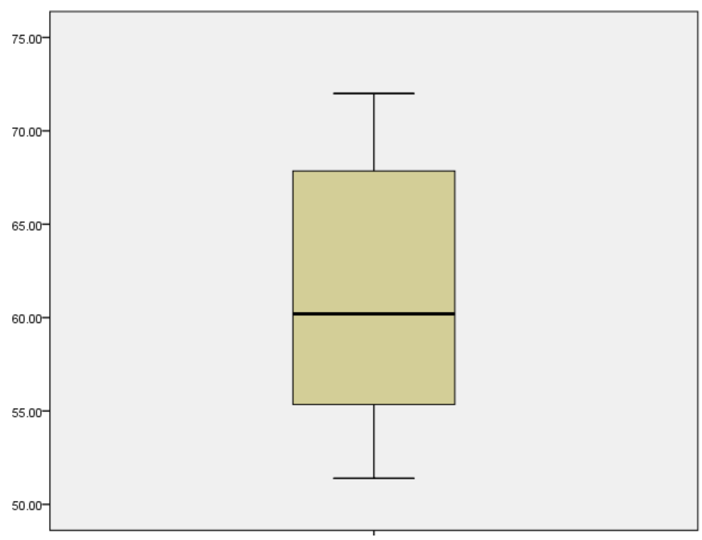


Figure 16: Boxplot of CBR of Bangladesh in 1980–2015

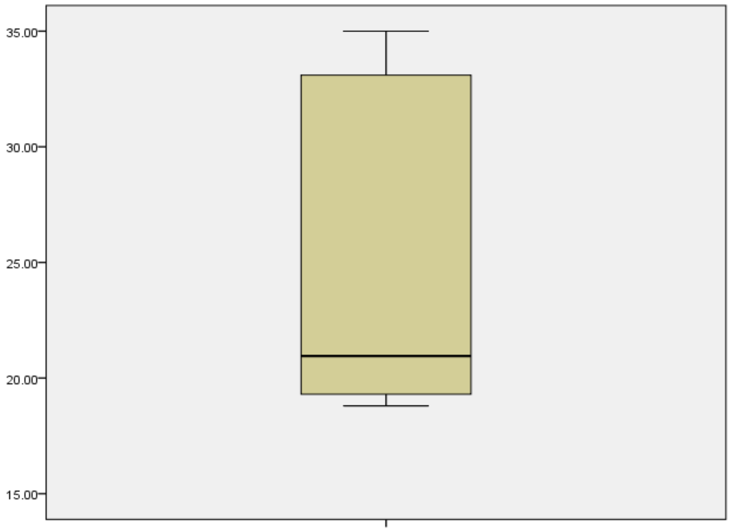


Figure 17: Boxplot of GRR of Bangladesh in 1980–2015

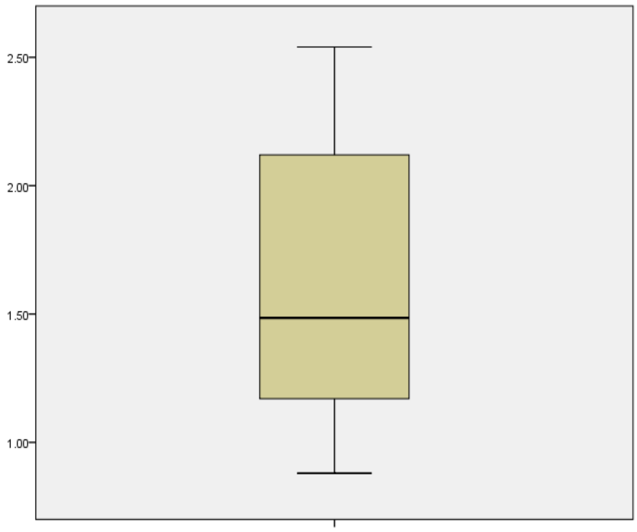
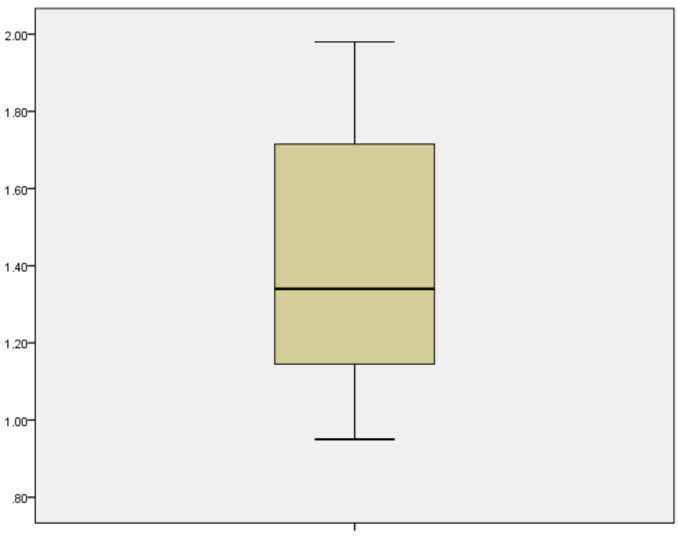


Figure 18: Boxplot of NRR of Bangladesh in 1980–2015



Stationarity Check

In this study, CDR, LEBM, LEBF, CBR, GRR, and NRR are plotted in graphs, reports of correlogram for autocorrelation and partial correlation, ADF, and PP tests show non-stationarity trends. The null hypothesis for ADF and PP tests shows a unit root. The descriptive and unit root test results for different indices are presented in Figures 19–24 and Table 3.

These findings are necessary for assessing the stationarity of data. These results suggest all variables may be stationary after performing difference or box-cox transformation. For better results of the econometric models, it is necessary and observed that CDR is also stationary after the first difference of box-cox at power -1 denoted as DICDR; LEBM, LEBF, GRR, and NRR also show stationarity after the first difference of box-cox at power -2 denoted by DSLEBM, DSLEBF, DSGRR, and DSNRR respectively.

Note that the box-cox of CDR at power -1 is $ICDR = 1 - \frac{1}{CDR}$, box-cox of LEBM, LEBF, GRR, and NRR at power -2 are $SLEBM = -\frac{1}{2}(-1 + \frac{1}{(LEBM)^2})$, $SLEBF = -\frac{1}{2}(-1 + \frac{1}{(LEBF)^2})$, $SGRR = -\frac{1}{2}(-1 + \frac{1}{(GRR)^2})$, and $SNRR = -\frac{1}{2}(-1 + \frac{1}{(NRR)^2})$ respectively.

Table 3: Results of Unit Root Tests

	ADF Test		PP Test	
	***t* value**	***p* value**	***t* value**	***p* value**
CDR	-1.423	0.8947	-0.6702	0.8413
LEBM	-0.2783	0.9182	-0.0918	0.9426
LEBF	-0.2271	0.9256	-0.2271	0.9256
CBR	-0.6382	0.8491	-0.7572	0.8187
GRR	-0.4835	0.8827	-0.4574	0.8878
NRR	-0.6247	0.8523	-0.6025	0.8574
DCDR	-4.4518	0.0012*	-4.4298	0.0013*
DLEBM	-5.3703	0.0001*	-7.9150	0.0000*
DLEBF	-6.9015	0.0000*	-6.9222	0.0000*
DCBR	-3.964	0.0044*	-3.913	0.0050*
DGRR	-6.4627	0.0000*	-6.4769	0.0000*
DNRR	-6.5356	0.0000*	-6.5821	0.0000*
DICDR	-4.426	0.0013*	-4.426	0.0013*
DSLEBM	-6.117	0.0000*	-9.667	0.0000*
DSLEBF	-8.230	0.0000*	-8.584	0.0000*
DSGRR	-4.818	0.0004*	-5.098	0.0002*
DSNRR	-3.539	0.0132*	-6.631	0.0000*

*Note: * indicates statistical significance at the 1% level.*

Figure 19: Trend of Level Data, 1st Difference, and 1st Difference of Box-cox Power -1 of CDR

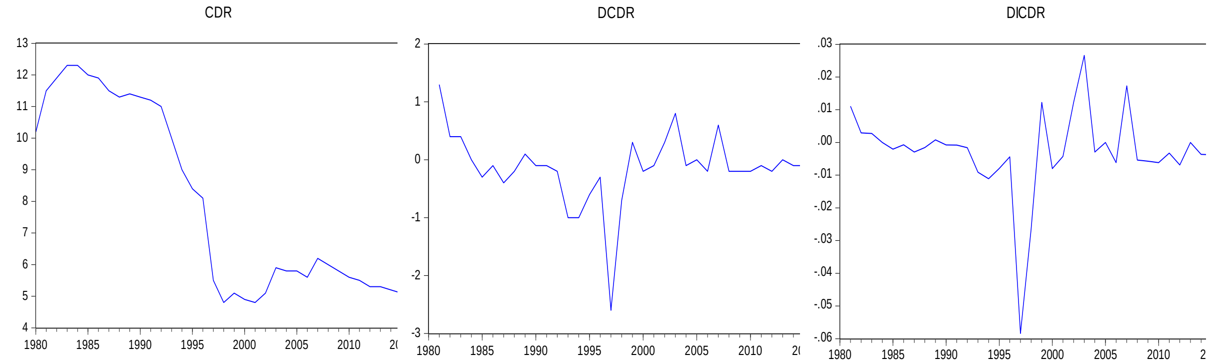


Figure 20: Trend of Level Data, 1st Difference, and 1st Difference of Box-cox Power -2 of LEBM

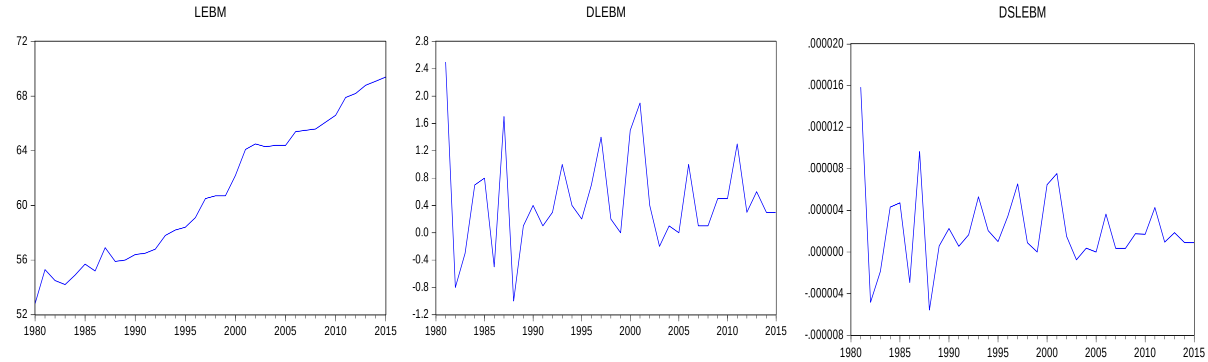


Figure 21: Trend of Level Data, 1st Difference, and 1st Difference of Box-cox Power -2 of LEBF

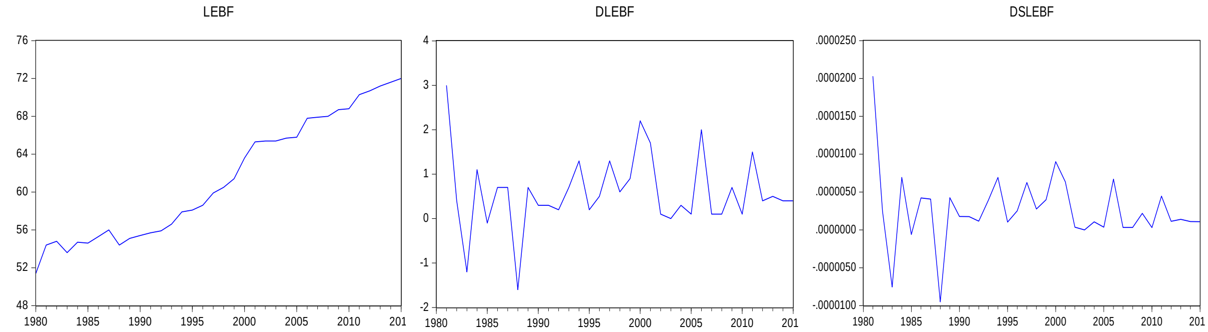


Figure 22: Trend of Level Data and 1st Difference of CBR

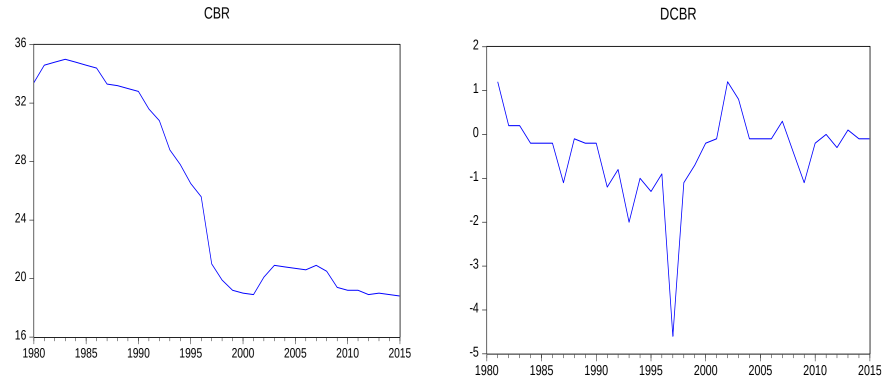


Figure 23: Trend of Level Data, 1st Difference, and 1st Difference of Box-cox Power -2 of GRR

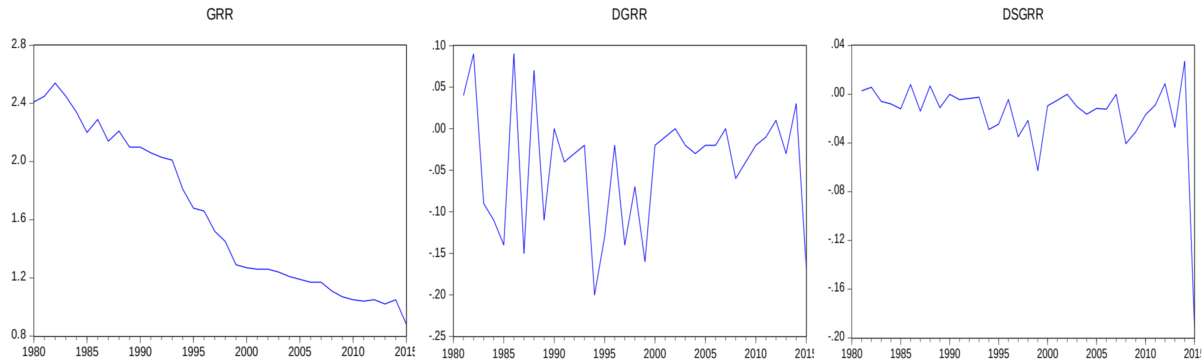
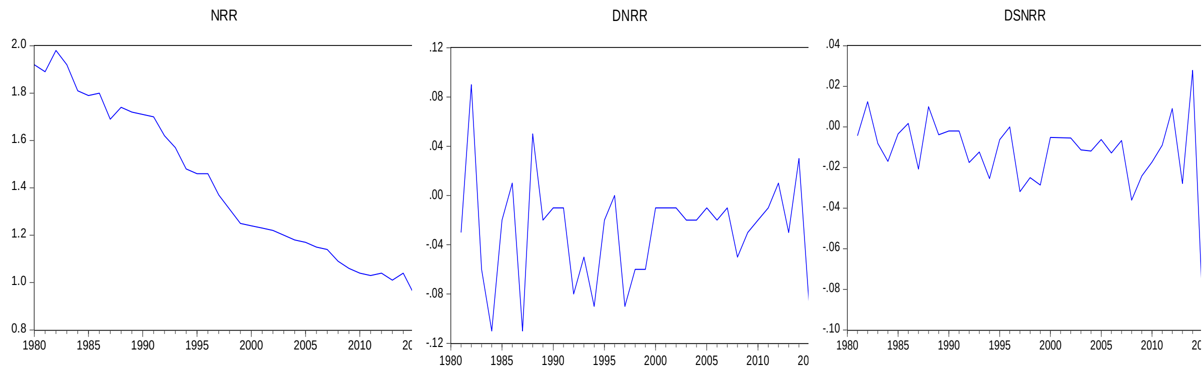


Figure 24: Trend of Level Data, 1st Difference, and 1st Difference of Box-cox Power -2 of NRR



Estimation and testing of models

The models are estimated based on an observed set of data, and then suitability has been tested. Various estimation procedures are used to know the suitability of the unknown parameters of the models. Usually, a suitable and appropriate model is selected from various formulations of econometric models. It shows that Autoregressive (AR) models have been fitted for all data sets separately. As AR models up to term lag 1 fitted for LEBM, GRR, and NRR, the AR model includes only term lag 2 for LEBF and CBR, and the AR model includes only term lag 6 for CDR. All models are well fitted based on t value and p value, and the developed models presented include values of the Akaike Information Criterion (AIC), Schwarz Information Criterion (SIC), and Hannan-Quinn (HQ) in Table 4.

The AR models fitted to the data were tested for adequacy through some analysis of the model's residuals. It shows correlograms for autocorrelation and partial correlation of residuals, and residual, actual, and fitted values of the models plotted in Figures 25–31. The Breusch-Pagan-Godfrey test is a Lagrange multiplier test of the null hypothesis of no heteroskedasticity (Breusch-Pagan, 1979; Godfrey, 1978).

The null hypothesis of the serial correlation of residuals test using the Breusch-Godfrey method is used, and no serial correlation in the residuals is shown up to the specified order. Results of the Heteroskedasticity Test and Serial Correlation Test of residuals, exhibited in Tables 5–6, indicate that the residuals of the models fitted are almost free from serial correlation and heteroskedasticity.

The AR models were developed separately for mortality measures such as CDR, LEBM, and LEBF and fertility measures like CBR, GRR, and NRR. Model validation techniques were also applied to check the validity of the fitted models, and this was discussed. The Statistical Package for Social Sciences (SPSS) and EViews were used to complete the different parts of the present study.

Table 4: Results of Fitted Models of CDR, LEBM, LEBF, CBR, GRR, and NRR.

	Coefficient	***t* value**	***p* value**	R^2	**Prob (F-statistic)**	**AIC**	**SIC**	**HQ**
For CDR: $DICDR_t = \alpha_0 + \alpha_6 DICDR_{t-6} + \varepsilon_t$ (*i*)				0.1040	0.0879***	-5.68	-5.59	-5.65
Constant	-0.004672	-1.8163	.0804***					
DICDR	-0.315191	-1.7703	.0880***					
For LEBM: $DSLEBM_t = \alpha_0 + \alpha_1 DSLEBM_{t-1} + \varepsilon_t$ (*ii*)				.1230	0.0419**	-22.50	-22.41	-22.47
Constant	0.0000023	3.9625	.0004*					
DSLEBM	-0.280476	-2.1188	.0420**					
For LEBF: $DSLEBF_t = \alpha_0 + \alpha_2 DSLEBF_{t-2} + \varepsilon_t$ (*iii*)				0.0868	0.0958***	-22.17	-22.08	-22.14
Constant	0.0000027	3.7951	.0006*					
DSLEBF	-0.224861	-1.7174	.0959***					
For CBR: $DCBR_t = \alpha_0 + \alpha_2 DCBR_{t-2} + \varepsilon_t$ (*iv*)				0.1144	0.0541**	2.73	2.83	2.77
Constant	-0.343962	-1.9581	.0593**					
DCBR	0.322865	2.0017	.0541**					
For GRR: $DSGRR_t = \alpha_0 + \alpha_1 DSGRR_{t-1} + \varepsilon_t$ (*v*)				0.1280	0.0559**	-8.36	-8.27	-8.33
Constant	0.001945	2.6415	.0127*					
DSGRR	0.081401	2.1674	.0377**					
For NRR: $DSNRR_t = \alpha_0 + \alpha_1 DSNRR_{t-1} + \varepsilon_t$ (*vi*)				0.1448	0.0263**	-5.10	-5.01	-5.07
Constant	-0.017441	-4.5068	.0001*					
DSNRR	-0.547818	-2.3282	.0264**					

*Note: *, **, and *** indicate statistical significance at the 1% level, 5% level, and 10% level, respectively.*

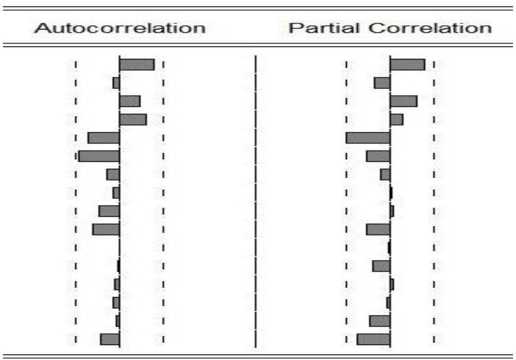
Table 5: Predicted CDR, LEBM, LEBF, CBR, GRR, and NRR from 2016 to 2030

Year	CDR	LEBM	LEBF	CBR	GRR	NRR
2016	5.02	70.10	73.01	18.68	0.92	0.97
2017	4.94	70.75	73.90	18.17	0.91	0.96
2018	4.85	71.41	74.82	17.67	0.90	0.95
2019	4.77	72.09	75.77	17.16	0.89	0.94
2020	4.69	72.80	76.77	16.65	0.88	0.93
2021	4.61	73.52	77.80	16.14	0.87	0.92
2022	4.54	74.27	78.87	15.63	0.86	0.91
2023	4.47	75.04	80.00	15.13	0.85	0.90
2024	4.40	75.84	81.17	14.62	0.85	0.89
2025	4.33	76.66	82.39	14.11	0.84	0.89
2026	4.26	77.51	83.67	13.60	0.83	0.88
2027	4.20	78.38	85.01	13.09	0.82	0.87
2028	4.14	79.29	86.42	12.59	0.81	0.86
2029	4.08	80.23	87.90	12.08	0.81	0.86
2030	4.02	81.20	89.46	11.57	0.80	0.85

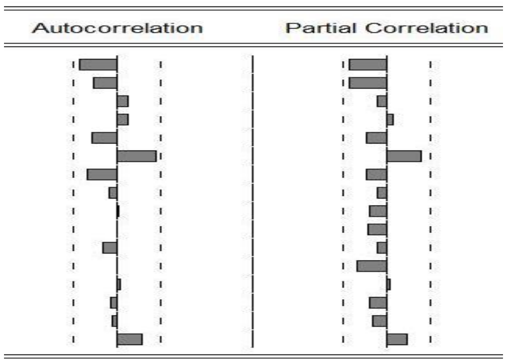
Table 6: Heteroskedasticity Test of Residuals using the Breusch-Pegan-Godfrey Method

	DICDR	DSLEBM	DSLEBF	DCBR	DSGRR	DSNRR
F-statistic	0.002367	0.000390	0.259322	1.518439	4.697910	4.087996
Prob(F-statistic)	0.9616	0.9844	0.6142	0.2271	0.0377	0.0516

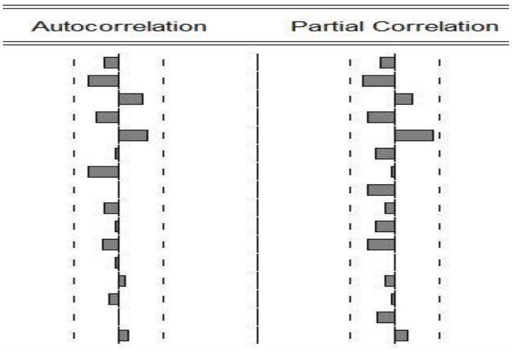
Figure 25: Correlogram of DICDR, DSLEBM, DSLEBF, DCBR, DSGRR, and DSNRR Respectively



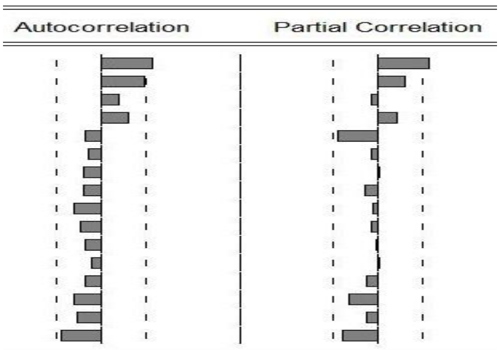
DICDR



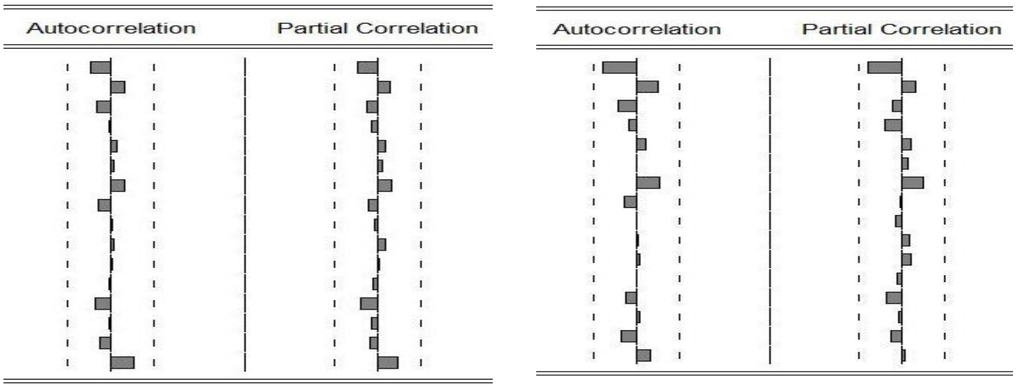
DSLEBM



DSLEBF



DCBR



DSGRR DSNRR

Figure 26: Actual, Fitted, and Residual Values of CDR for Model (i)

Figure 27: Actual, Fitted, and Residual Values of LEBM for Model (ii)

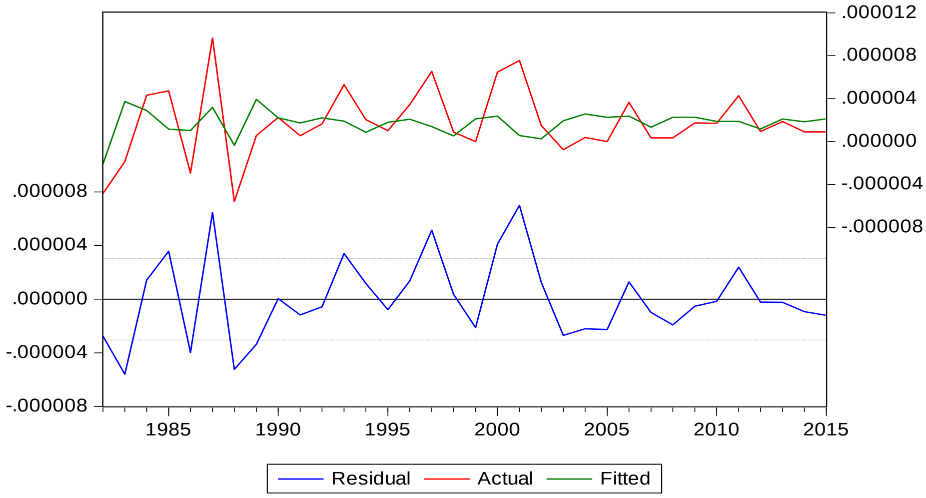


Figure 28: Actual, Fitted, and Residual Values of LEBF for Model (iii)

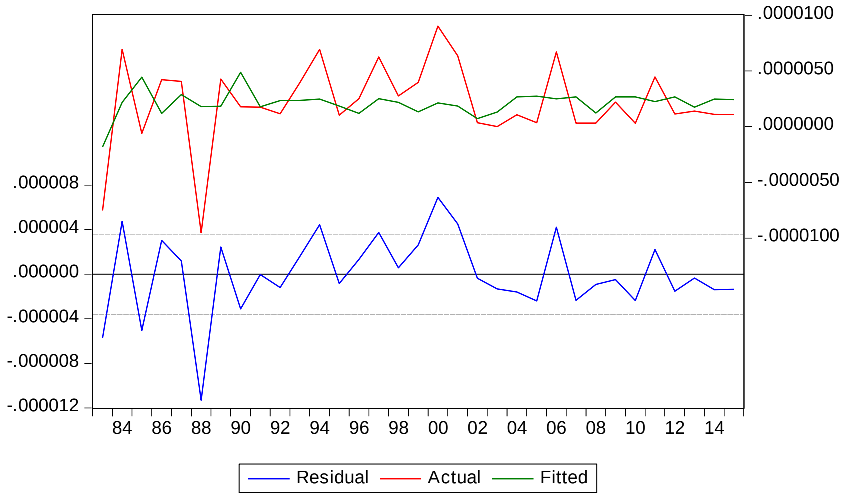


Figure 29: Actual, Fitted, and Residual Values of CBR for Model (iv)

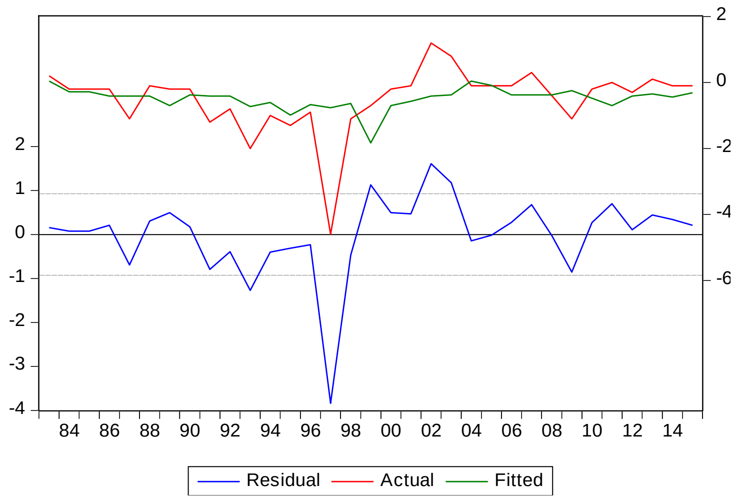


Figure 30: Actual, Fitted, and Residual Values of GRR for Model (v)

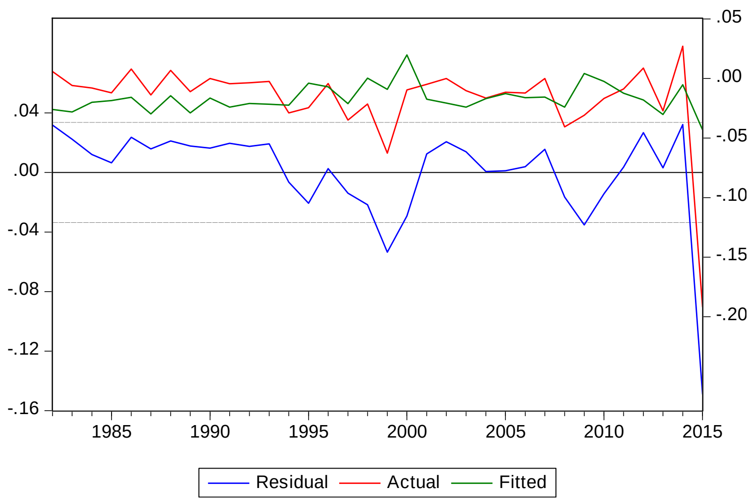
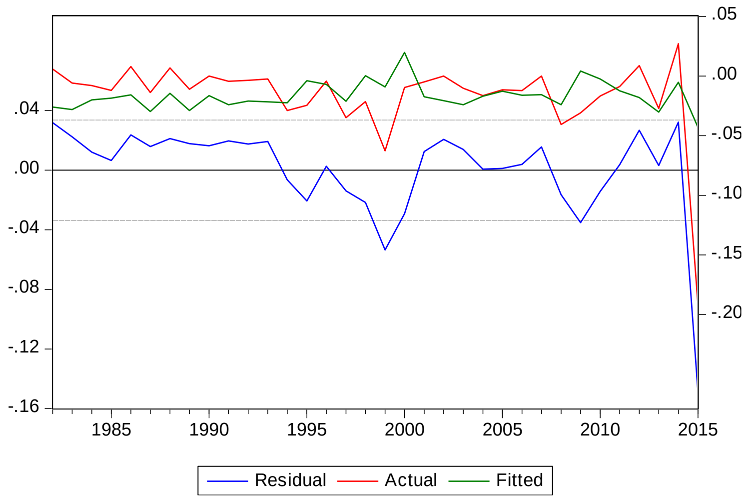


Figure 31: Actual, Fitted, and Residual Values of NRR for Model (vi)



Validity of forecasted values

Root Mean Squared Error (RMSE) measures the validity of forecasted values by calculating the average difference between a model's predicted values and the actual values. RMSE is usually used for numerical predictions (Christie & Neill, 2022; Chu & Shirmohammadi, 2004; Hodson, 2022; Singh et al., 2004; Vazquez-Amábile & Engel, 2005).

$$RMSE = \sqrt{\frac{1}{n}\sum_{i}^{n}(Y_i - P_i)^2}$$

Where Yi are the actual values, Pi are the predicted values of a variable, and n is the number of observations available for analysis. RMSE's best value is 0, and the worst value is +∞ (Chicco et al., 2021). RMSE of CDR, LEBM, LEBF, CBR, GRR, and NRR using the actual and forecasted data from 2016 to 2019 were estimated as 0.13, 0.59, 0.95, 0.58, 0.11, and 0.04, respectively. This RMSE's values show the strong acceptance of forecasted CDR, GRR, and NRR and overall acceptance of LEBM and CBR, excluding LEBF.

Results and discussion

R models have been chosen and fitted for CDR, LEBM, LEBF, CBR, GRR, and NRR based on p value, AIC, SIC, and HQ presented in Table 4. AR models up to term lag 1 fitted for LEBM, GRR, and NRR, AR model including only term lag 2 for LEBF and CBR, and AR model including only term lag 6 for CDR. In these models, constant terms and coefficients are significant, from a confidence level of 1% to 10%, as shown in Table 4.

It is necessary to check the residuals of the fitted models to ensure they are well-fitted. For this, the results of the heteroskedasticity test using the Breusch-Pagan-Godfrey Method and Serial Correlation using the Breusch-Godfrey Method of Residuals are shown in Tables 6–7 and 25–31. No serial correlation and heteroskedasticity were observed among the residuals of the models except DSGRR and DSNRR.

Using the fitted models, CDR, LEBM, LEBF, CBR, GRR, and NRR have been forecasted from 2016 to 2030 and presented in Table 5. CDR per 1000 population, LEBM, LEBF, CBR, GRR and NRR of Bangladesh from 2016 to 2019 is estimated as 5.02, 4.94, 4.85 and 4.77; 70.10, 70.75, 71.41 and 72.09; 73.01, 73.90, 74.82 and 75.77; 18.68, 18.17, 17.67 and 17.16; 0.92, 0.91, 0.90 and 0.89; 0.97, 0.96, 0.95 and 0.94 respectively whereas actual value is observed in Bangladesh Bureau of Statistics (2023) as 5.1, 5.1, 5.0 and 4.9; 70.3, 70.6, 70.8 and 71.1; 72.9, 73.5, 73.8 and 74.2; 18.7, 18.5, 18.3, and 18.1; 1.02, 1.02, 1.00 and 1.00; 1.00, 1.00, 0.99 and 1.00 respectively presented in Tables 5 and 1.

The United Nations (2024) showed Bangladesh's CDR per 1000 population, LEBM, LEBF, and CBR in 2016 and 2030 as 5.6 and 5.5, 69.08 and 73.53, 73.32 and 78.16, and 19.0 and 15.2, respectively. The CDR and CBR per 1000 population of Bangladesh in 2030 are forecasted as 4.02 and 11.57, respectively. GRR, NRR, LEBM, and LEBF of Bangladesh in 2030 are also forecasted as 0.80, 0.85, 81.20, and 89.46, respectively.

Forecasted values from 2016 to 2030 in Table 5 show that CDR, CBR, GRR, and NRR trends decrease while LEBM and LEBF increase over time. It is also observed from forecasted values that CDR, GRR, and NRR are decreasing slowly, and CBR is decreasing in a very downtrend. Decreasing CDR, GRR, NRR, and CBR is good for life expectancy for male and female populations because they are negatively correlated.

From the forecasted values, it is estimated that LEBM and LEBF will increase in the future, helping Bangladesh achieve its sustainable development goals. Bangladesh is an overpopulated country, and its population has been on an uptrend over time (Bangladesh Bureau of Statistics, 2022). However, decreasing CDR, GRR, NRR, and CBR over time indicates that the population will experience a downtrend. Increasing LEBM and LEBF over time shows that Bangladesh's aged population will increase.

Table 7: Serial Correlation of Residuals Test using the Breusch-Godfrey Method

	DICDR	DSLEBM	DSLEBF	DCBR	DSGRR	DSNRR
F-statistic	0.829181	0.000313	0.982894	2.582030	2.091766	1.766580
Prob(F-statistic)	0.3709	0.9860	0.3294	0.1186	0.1581	0.1935

Limitations of the study

This study used autoregressive models for trend analysis and forecasting of the above-mentioned time series variables with 36 observations. The number of observations was expected to be high enough to fit autoregressive models. In this study, Autoregressive models were developed based on *t* values, *p* values, AIC, SIC, and HQ, neglecting the values of R^2 and the Heteroskedasticity test of residuals of DSGRR and DSNRR.

Conclusions

Demographic parameters ultimately describe the health status of Bangladesh. AR models have been fitted for CDR, LEBM, LEBF, CBR, GRR, and NRR using time series data from 1980-2015 and forecasted mortality measures such as CDR, LEBM, and LEBF, and fertility measures such as CBR, GRR, and NRR up to 2030. After 2015, CDR, CBR, GRR, and NRR decreased over time, while LEBM and LEBF increased. CDR, CBR, GRR, and NRR of Bangladesh in 2030 will decrease at the rate of 21.18%, 38.46%, 9.1%, and 10.53%, respectively, compared to 2015, while LEBM and LEBF will increase at the rate of 17.0% and 24.25% respectively. The trend of these parameters shows the positive health status of Bangladesh over time.

However, to make this trend sustainable, several decisions have to be taken over time by the government, non-government organizations, and policymakers, such as increasing educational institutions, confirming immunization programs for infants, assuring healthcare facilities, providing effective programs, and special preparation to verify all types of needs of the aged population. Any interested researcher or organization may conduct future research to examine the relationship between mortality and fertility parameters.

References

Aksoy, Y., Basso, H. S., Smith, R. P., & Grasl, T. (2019). Demographic structure and macroeconomic trends. *American Economic Journal: Macroeconomics*, *11*(1), 193–222. http://dx.doi.org/*10.1257/mac.20170114*

Balbo, N., Billari, F. C., & Mills, M. (2013). Fertility in advanced societies: A review of research. *European Journal of Population*, *29*, 1–38. http://dx.doi.org/10.1007/s10680-012-9277-y

Bangladesh Bureau of Statistics. (1989–2012). *Statistical Yearbook of Bangladesh* (Editions 1988–2011). Government of the People's Republic of Bangladesh.

Bangladesh Bureau of Statistics. (2013–2023). *Statistical Yearbook of Bangladesh* (Editions 2012–2021). Government of the People's Republic of Bangladesh. https://bbs.gov.bd/site/page/29855dc1-f2b4-4dc0-9073-f692361112da/Statistical-Yearbook

Beg, A. B. M. R. A., & Islam, M. R. (2016). Modeling and forecasting population growth of Bangladesh. *American Journal of Mathematics and Statistics*, *6*(4), 190–195. http://article.sapub.org/10.5923.j.ajms.20160604.08.html

Begum, S. (1990). Population birth, death and growth rate in Bangladesh: Census estimates. *Bangladesh Development Studies*, *18*(2), 51–75. http://www.jstor.org/stable/40795379

Breusch, T. S., & Pagan, A. R. (1979). A simple test for heteroskedasticity and random coefficient variation. *Econometrica. 47*(5), 1287–1294. http://dx.doi.org/10.2307/1911963

Chicco, D., Warrens, M. J., & Jurman, G. (2021). The coefficient of determination R-squared is more informative than SMAPE, MAE, MAPE, MSE and RMSE in regression analysis evaluation. *PeerJ. Computer Science*, *7*, Article e623. https://doi.org/10.7717/peerj-cs.623

Christie, D., & Neill, S. P. (2022). Measuring and observing the ocean's renewable energy resource. In J. Yan (Ed.), *Comprehensive renewable energy* (2nd ed., pp. 149–175). Elsevier.

Chu, T. W., & Shirmohammadi, A. (2004). Evaluation of the SWAT model's hydrology component in the Piedmont physiographic region of Maryland. *Transactions of the ASAE, 47*(4), 1057–1073. https://doi.org/10.13031/2013.16579

Denton, F. T., Feaver, C. H., & Spencer, B. G. (2005). Time series analysis and stochastic forecasting: An econometric study of mortality and life expectancy. *Journal of Population Economics*, *18*, 203–227. https://doi.org/10.1007/s00148-005-0229-2

Godfrey, L. G. (1978). Testing against general autoregressive and moving average error models when the regressors include lagged dependent variables. *Econometrica, 46*(6), 1293–1302. https://doi.org/10.2307/1913829

Goldstein, J. R., Sobotka, T., & Jasilioniene, A. (2009). The end of "lowest-low" fertility? *Population and Development Review*, *35*(4), 663–699. https://doi.org/10.1111/j.1728-4457.2009.00304.x

Guets, W., & Behera, D. K. (2022). Does disability increase households' health financial risk: evidence from the Uganda demographic and health survey. *Global Health Research and Policy*, *7*, Article 2. https://doi.org/10.1186/s41256-021-00235-x

Halicioglu, F. (2011). Modeling life expectancy in Turkey. *Economic modelling*, *28*(5), 2075–2082. https://doi.org/10.1016/j.econmod.2011.05.002

Hodson, T. O. (2022). Root-mean-square error (RMSE) or mean absolute error (MAE): When to use them or not. *Geoscientific Model Development*, *15*(14), 5481–5487. https://doi.org/10.5194/gmd-15-5481-2022

Islam, M. R., Islam, M. N., Ali, M. K., & Mondol, M. N. I. (2005). Indirect estimation and mathematical modeling of some demographic parameters of Bangladesh. *Oriental Anthropologist*, *5*(2), 163–171. https://doi.org/10.1177/0976343020050203

Kelley, A. C., & Schmidt, R. M. (2005). Evolution of recent economic-demographic modeling: A synthesis. *Journal of Population Economics*, *18*, 275–300. https://doi.org/10.1007/s00148-005-0222-9

Maddala, G. S. (1992). *Introduction to econometrics* (2nd Ed.). MacMillan.

Mannan, M. I., Rahman, M. S., & Islam, M. S. (2014). Importance to construct Indigenous mortality table from crude death rate for the life insurance industry in Bangladesh. *Journal of Economics and Finance*, *5*(6), 15–18. https://doi.org/10.9790/5933-05611518

Matthews, S. A., & Parker, D. M. (2013). Progress in spatial demography. *Demographic Research*, *28*, 271–312. https://doi.org/10.4054%2Fdemres.2013.28.10

Molugaram, K., & Rao, G. S. (2017). Chapter 12 - Analysis of time series. In K. Molugaram & G. S. Rao (Eds.), *Statistical Techniques for Transportation Engineering* (pp. 463–489). Butterworth-Heinemann. https://doi.org/10.1016/B978-0-12-811555-8.00012-X

Muyeed, A., Siddiqi, M. N. A., & Tawabunnahar, M. (2020). Prevalence and severity of COVID-19 disease in Bangladesh: A trend analysis. *Journal of Health & Biological Sciences*, *8*(1), 1–8. https://doi.org/10.12662/2317-3076jhbs.v8i1.3285.p1-8.2020

Nandi, D. C., Hossain, M. F., Roy, P., & Ullah, M. S. (2023). An investigation of the relation between life expectancy and socioeconomic variables using path analysis for Sustainable Development Goals (SDG) in Bangladesh. *PLOS ONE*, *18*(2), Article e0275431. https://doi.org/10.1371/journal.pone.0275431

Rahman, M. M., & Alam, K. (2023). The role of socio-economic and female indicators on child mortality rate in Bangladesh: A time series analysis. *Omega*, *86*(3), 889–912. https://doi.org/10.1177/0030222821993616

Robinson, J. G., & Jensen, E. B. (2020). A demographic evaluation of the stability of American Community Survey (ACS) estimates for ACS test sites: 2000 to 2011. In J. Singelmann & D. Poston Jr. (Eds.), *Developments in Demography in the 21st Century* (The Springer Series on Demographic Methods and Population Analysis, Vol. 48, pp. 25–39). Springer, Cham. https://doi.org/10.1007/978-3-030-26492-5_3

Rubi, M. A., Bijoy, H. I., & Bitto, A. K. (2021). Life expectancy prediction based on GDP and population size of Bangladesh using multiple linear regression and ANN model. *12th International Conference on Computing Communication and Networking Technologies* (pp. 1–6). https://doi.org/10.1109/ICCCNT51525.2021.9579594

Salvati, L., Benassi, F., Miccoli, S., Rabiei-Dastjerdi, H., & Matthews, S. A. (2020). Spatial variability of total fertility rate and crude birth rate in a low-fertility country: Patterns and trends in regional and local scale heterogeneity across Italy, 2002–2018. *Applied Geography*, *124*, Article 102321. https://doi.org/10.1016/j.apgeog.2020.102321

Sam, C. Y., McNown, R., & Goh, S. K. (2019). An augmented autoregressive distributed lag bounds test for cointegration. *Economic Modelling*, *80*, 130–141. https://doi.org/10.1016/j.econmod.2018.11.001

Senturk, I., & Ali, A. (2021). Socioeconomic determinants of gender-specific life expectancy in Turkey: A time series analysis. *Sosyoekonomi,* *29*(49), 85–111. https://doi.org/10.17233/sosyoekonomi.2021.03.05

Shen, T., Lazzari, E., & Canudas-Romo, V. (2023). The contribution of survival to changes in the net reproduction rate. *Population Studies*, *77*(2), 163–178. https://doi.org/10.1080/00324728.2023.2187441

Shubat, O., & Bagirova, A. (2022). Forecasting the length of grandparenthood with limited information resources: Evidence from Russia. *Journal of Population and Social Studies*, *30*, 251–268. https://so03.tci-thaijo.org/index.php/jpss/article/view/257752

Singh, J., Knapp, H. V., & Demissie, M. (2004). Hydrologic modeling of the Iroquois River watershed using HSPF and SWAT. *ISWS CR 2004-08*. Illinois State Water Survey. https://swat.tamu.edu/media/90101/singh.pdf

United Nations. (2024). *World Population Prospects 2024.* https://population.un.org/wpp/

United Nations Development Programme (UNDP). (1991). *Human Development Report 1991*. https://hdr.undp.org/system/files/documents/hdr1991encompletenostatspdf.pdf

Vazquez-Amábile, G. G., & Engel, B. A. (2005). Use of SWAT to compute groundwater table depth and streamflow in the Muscatatuck River watershed. *Transactions of the ASAE, 48*(3), 991–1003. https://doi.org/10.13031/2013.18511

Voss, P. R. (2007). Demography as a spatial social science. *Population Research and Policy Review*, *26*, 457–476. https://doi.org/10.1007/s11113-007-9047-4

Winkelmann, R., & Zimmermann, K. F. (1994). Count data models for demographic data. *Mathematical Population Studies*, *4*(3), 205–221. https://doi.org/10.1080/08898489409525374

Xie, Y. (2000). Demography: Past, present, and future. *Journal of the American Statistical Association*, *95*(450), 670–673. https://doi.org/10.1080/01621459.2000.10474248

Yang, H., Wang, S., Ren, Z., Liu, H., Tong, Y., & Wang, N. (2022). Life expectancy, air pollution, and socioeconomic factors: A multivariate time-series analysis of Beijing City, China. *Social Indicators Research*, *162*(3), 979–994. https://doi.org/10.1007/s11205-019-02162-4