

## วิธีวิวัฒนาการโดยใช้ผลต่างสำหรับการแก้ไขปัญหาการมอบหมายงาน แบบหลายลำดับชั้น: กรณีศึกษาการขนส่งไก่

### DIFFERENTIAL EVOLUTION ALGORITHM FOR SOLVING MULTI-STAGES ASSIGNMENT PROBLEM: A CASE STUDY OF CHICKEN TRANSPORTATION

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#### บทคัดย่อ

ปัญหาการมอบหมายงานคือปัญหาในการมอบหมายให้พนักงานคนหนึ่งไปทำงานงานหนึ่ง การมอบหมายงานในแต่ละครั้งจะเกิดต้นทุนในการมอบหมายงานที่แตกต่างกัน เป้าหมายในการแก้ไขปัญหาการมอบหมายงานคือจะมอบหมายงานอย่างไรถึงจะทำให้ต้นทุนในการมอบหมายงานต่ำสุด ปัญหาการมอบหมายงานแบบหลายลำดับชั้นจะมีความซับซ้อนมากกว่าปัญหาการมอบหมายงานแบบดั้งเดิม ลำดับที่หนึ่งคือการมอบหมายฟาร์มไก่ให้กับฟาร์มไข่พร้อมกับปริมาณที่ทำการขนส่ง ลำดับที่สองมอบหมายประเภทของรถบรรทุกที่เหมาะสมสำหรับการขนส่งดังกล่าว การมอบหมายงานทั้งสองลำดับชั้นจะเกิดขึ้นพร้อมกันดังนั้นต้นทุนในการมอบหมายงานจะขึ้นกับการตัดสินใจในการมอบหมายนั้น

วิธีการแมนตรงเป็นวิธีการที่ดีที่สุดในการหาคำตอบสำหรับปัญหาการมอบหมายงาน ยกตัวอย่างเช่น Simplex method, the Hungarian method เป็นต้น เริ่มต้นทำการศึกษาจากการพัฒนาแบบจำลองทางคณิตศาสตร์และการพัฒนาโปรแกรม LINGO เพื่อหาคำตอบที่ดีที่สุด ถึงแม้ว่าวิธีการแมนตรงจะได้คำตอบที่ดีที่สุดแต่อาจใช้เวลานานในการหาคำตอบ วิธีการฮิวริสติกส์เป็นวิธีการทางเลือกในการแก้ปัญหโดยใช้เวลาในการหาคำตอบไม่นาน อย่างไรก็ตามคำตอบที่ได้จะใกล้เคียงกับคำตอบที่ดีที่สุด ในกรณีศึกษาใช้วิธีวิวัฒนาการโดยใช้ผลต่าง (DE) เริ่มจากการกำหนดตัวอย่างเริ่มต้น หลังจากนั้นก็ทำซ้ำขั้นตอน Mutation, Recombination, และ Selection ไปเรื่อย ๆ จนกว่าจะถึงเงื่อนไขที่กำหนด

ผู้วิจัยทำการศึกษาโดยการพัฒนาแบบจำลองทางคณิตศาสตร์ที่ประกอบไปด้วยฟังก์ชันหลักและข้อจำกัดทั้งหมด หลังจากนั้นทำการพัฒนาโปรแกรมลิงโก แล้วทำการทดสอบด้วยการสร้างกลุ่มตัวอย่างที่แตกต่างกันจากขนาดของกลุ่มตัวอย่าง ในแต่ละกลุ่มจะมีจำนวน 5 ตัวอย่าง เมื่อทำการทดสอบแบบจำลองเรียบร้อยแล้วก็จะทำการใช้ข้อมูลจากกรณีศึกษา ในขณะเดียวกันก็ทำการพัฒนาวิธีวิวัฒนาการโดยใช้ผลต่าง โดยใช้กลุ่มตัวอย่างที่สร้างขึ้นข้างต้น และข้อมูลจากกรณีศึกษาเป็นลำดับถัดไป

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วิธีการแม้นตรงสามารถหาคำตอบได้ในกรณีศึกษาที่มีขนาดเล็กและขนาดกลาง DE สามารถหาคำตอบได้ในเวลา 0.2 วินาที ในขณะที่โปรแกรมลิงโกสามารถหาคำตอบได้ในเวลา 5 วินาที หากดัชนีเท่ากับ 100 หมายความว่า DE ให้คำตอบเท่ากับวิธีการแม้นตรง สำหรับกรณีตัวอย่างขนาดเล็กมีดัชนีเท่ากับ 118.4 อาจเนื่องจากระยะทางระหว่างแต่ละฟาร์มมีค่าสูง ในกรณีตัวอย่างขนาดกลาง DE ใช้เวลา 0.6 วินาที ส่วนโปรแกรมลิงโกใช้เวลา 103.6 วินาทีในการหาคำตอบ โดยมีดัชนีเท่ากับ 106.9 ซึ่งถือว่าใกล้เคียงคำตอบที่ดีที่สุดมากกว่ากลุ่มตัวอย่างขนาดเล็ก สำหรับกลุ่มตัวอย่างขนาดใหญ่และกรณีศึกษาไม่สามารถหาคำตอบที่ดีที่สุดได้ในเวลาที่เหมาะสม แต่อย่างไรก็ตามสามารถหาคำตอบที่เป็นไปได้ภายในเวลา 72 ชั่วโมง ดังนั้นจึงทำการกำหนดเวลาในการหาคำตอบสำหรับโปรแกรมลิงโกสำหรับปัญหาขนาดใหญ่และกรณีศึกษา มีผลการศึกษาดังนี้ กลุ่มตัวอย่างขนาดใหญ่มีดัชนี 107.1 และใช้เวลาในการหาคำตอบ 4.2 วินาที สำหรับกรณีศึกษามีดัชนีเท่ากับ 109.0 โดยใช้เวลา 7.8 วินาที

เพื่อเป็นการพัฒนาคำตอบสำหรับวิธี DE งานวิจัยในอนาคตควรมีการปรับปรุงวิธีวิวัฒนาการโดยใช้ผลต่างเพื่อให้ได้คำตอบที่ดียิ่งขึ้น นอกจากนี้อาจพัฒนาวิธีวิวิธวิธีอื่น ๆ เช่น Particle Swarm Optimization (PSO) หรือวิธีอื่น ๆ ที่จะมาใช้แทนวิธีการ DE

**คำสำคัญ:** ปัญหาการมอบหมายงาน ปัญหาการมอบหมายงานแบบหลายชั้น วิธีวิวัฒนาการโดยใช้ผลต่าง

### Abstract

The assignment problem is a problem of assigning an agent to one job. Any matching of assignment come with a cost. The minimize total cost is the goal of assignment problem. The multi-stage assignment problem is more complicated than the classical one. Stage one is assign chicken's farm to egg's farm then quantity demanded is created. Stage two, type of truck that matches quantity demanded is assigned. All of the assignments must be assigned in one decision. Thus the costs of assignment depend on that decisions.

The exact method is the best way for solving assignment problem for instances simplex method, the Hungarian method, and etc. It is started by developing a mathematical model and LINGO program. Not only exact method that can be solved assignment problem. The heuristic method is an alternative way to solve the problem. In this study, Differential Evolution (DE) is selected. After the initial solution is set, DE will do mutation, recombination, and selection, repeatedly. The end condition in this study is a number of iterations.

We develop a mathematical model that include fitness function and all restrictions. After that, we develop LINGO program and test it by generating 3 groups of sample by using the size of sample and each group has 5 samples. Afterward, we use this case study data instead of groups of sample. At the same time, we develop DE algorithm and test with those groups of sample and case study data.

For small and medium size, we find the optimal solution. DE find the best solution for small size within 0.2 seconds while LINGO time is around 5 seconds. DE index is 118.4 on average that might not look good because of the generated distance in that sample. For medium size, DE time is only 0.6 seconds while LINGO time is 103.6 seconds on average. The index of DE is 106.9 on average. It means the results move closer to the best answer. We cannot find the optimal solution for large size and case study data within appropriated time however we can find a feasible solution within 72 hours. Therefore, we run LINGO for 72 hours and use it for both large size and case study. An average index of large size is 107.1 and DE time is only 4.2 seconds on average. The case study DE time is 7.8 on average while an index is 109.0.

To improve DE result, the future research should modify DE hence result should be closer to the best answer. The new heuristics which might be Particle Swarm Optimization (PSO), or any others heuristics should be used instead of DE.

**Keywords:** Assignment Problem, Multi-Stage Assignment Problem, Differential Evolution

## Introduction

The transportation in this case study is a chicken transportation from chicken farm to egg farm. There are 4 types of truck which are ten-wheel truck, eight-wheel truck, modified four-wheel truck and ordinary four-wheel truck. Each type of truck has different capacity moreover each hen's farm has different in quantity supplied as same as each egg's farm also have different in quantity demanded. Previously, the manager uses First in First Out method (FIFO), for instances; the first truck in queue will be used first. In the case study, we classify this problem as multi-stages assignment problem. The first stage, we assign hen's farm to egg's farm. The second stage, we assign truck that approximately matches quantity demanded and quantity supplied. If a truck cannot transport all chickens in one round, then round 2, or round 3 are applied. In each round, type of truck, hen's farm and egg's farm have to be the same.

The goal of solving assignment problem is how to make an assignment with the lowest cost. In this case study, we have 2 types of assignment cost which are transportation cost and opportunity cost. Transportation cost is the major assignment cost while opportunity cost fulfills cost effective method. When trucks do not transport with full capacity, the opportunity cost will be applied. Transportation cost depends mainly on the distance between hen farm and egg farm and fuel consumption of each type of truck. The goal of this study is to minimize both transportation and opportunity cost.

This problem will solve by 2 directions; exact and heuristic method. The exact method will start by generating a mathematical model of case study and using LINGO to find the best solution. It is a time-consuming method. The alternative method that can find an appropriate answer which is called “Heuristics method”. The heuristic method will discovery acceptable result and spend less time than exact method. The selected heuristics is Differential Evolution (DE) because DE expands searching area that might be close to the optimal solution. The process of DE start with encoding decoding and initial population then mutation, recombination, and selection. DE compute fitness function only in selection process that can reduce time-consuming while another heuristic method may compute fitness function in every step.

In this study, we will classify into 4 parts. Part I is an introduction. Part II is a review of assignment problem. Part III is a case study. Part IV is mathematical model and restrictions for a case study. Part V is Differential Evolution (DE). Part VI is a result from the case study. And part VII is the conclusion of a case study.

## Review literature

Transportation problem goal is how to make a transportation from town to town that has minimum cost. It might be called Assignment Problem (AP) (Monge, 1784). Simple assignment problem is solved by bipartite matching. It is introduced by Frobenius (1912-1917) and Konig (1915-1931). The simplex method that is used to solve this problem was introduced by Dantzig (1951). According to the complex of the simplex method, Khun (1955) presented Hungarian method as a new method to solve assignment problem. Ford and Fulkerson (1956) confirmed that the Hungarian method is one of the fastest ways to solve assignment problem. The problem size is one of the major factors that determine assignment problem.

Combinatorial optimization is the study that finds out the optimal solution from a finite set of items. The feasible solutions are discrete or can be reduced to discrete. The objective is to find the best solution. There are some common problems which are Travel Salesman Problem (TSP), Minimum Spanning Tree Problem (MST), or Assignment Problem (AP) and so on. Assignment problem is one of the most famous problems in this area. The first use of assignment problem was introduced by Votaw and Orden (1952) refer to Pentico (2005). This problem is one to one basis means one agent is assigned to only one job, and vice versa. A number of agents and jobs must be the same for the classical assignment problem. There are variations of assignment problem. The dummy can be added to the minimum of agents or jobs that meet the requirements. Kuhn (1955) present the Hungarian method as a solution for the classical assignment problem.

Fulkerson, Glicksberg, and Gross (1953) present the first Bottleneck Assignment Problem (BAP). It is a variation of assignment problem that minimized the maximum problem or maximized the minimum problem. BAP is the problem of how to transport a rotten product from depots to markets without wastes. If time is an important factor that turns fresh goods into decay, then BAP is the problem of how to minimized transportation time for maximum delivered products from depots to markets. This is close to this study that is how to transport hens from hen's farm to egg's farm with shortage time to avoid plague or unhealthy of hen during transportation.

The relaxation on assumptions of AP can be made AP move closer to the real world. The Generalized Assignment Problem (GAP) was firstly introduced by Ross and Soland (1975). GAP is the problem that assigns  $m$  agents to  $n$  jobs. Once assign an agent for a job, it consumes resources. The assignment must not exceed job's capacity. The differences from previous assignment are 1) the number of agents and the number of jobs might not be the same, 2) one assignment uses resources and 3) jobs has the capacity. In this study, the amount of hens in transportation (resource use) should be less than or equal to quantity demanded of Egg's farm. The number of Hen's farm and Egg's farm are not the same.

GAP can be considered as the Weighted Assignment Problem (WAP) (Ross & Zoltners, 1979). WAP discover the optimal solution of assigning an agent with recognized performance to a task. The binary variable for WAP is  $x_{ijk}$  is 1 when assign task  $j$  to agent  $i$  with performance level  $k$ . All agent are human thus there are an upper limit and lower limit for them. The decision variable ( $x_{ijk}$ ) of the WAP show that the choice is made at the same time for assigning agent  $i$ , task  $j$  and performance level  $k$ .

There are many algorithms for the classical assignment problem. The most wanted method is an exact algorithm that can find the optimal solution. There are many exact algorithms to solve for assignment problems such as simplex method, branch and bound, the Hungarian method, and many methods. The Hungarian Method (Kuhn 1955) is one of the most famous methods. This method is perfect for small sized problem however big sized problem this method would take a long time to compute or might be very long time to solve. This method may not apply to variations of assignment problem. An answer from the exact method guarantees for an optimal solution.

The exact method may replace by an alternative method which is a heuristic method. The branch and bound algorithm as an exact method is used to solve general quadratic assignment problem for only up to 10 sample size because of time-consuming (Kaku & Thompson, 1983). The exact method, branch and bound, is used to solve GAP and improved by using the variable fixing method (Posta, Ferland, & Michelon, 2011). This study has 10 agents

and 100 jobs for a sample and time limit to 24 hours of running time. The size of population and complex of problem are the major factors that cause time-consuming in the exact method process.

Differential Evolution (DE) is an alternative method that optimizes a problem by improving a solution repeatedly. DE has 4 steps which are 1) initial solution, 2) Mutation, 3) Recombination and 4) Selection. DE first used by Storn and Price (1997). DE is used to solve many problems such as Vehicle Routing Problem (VRP), Assembly Line Balancing, Generalized Assignment Problem (GAP) etc. Ordinary DE has no local search that why DE work faster than another algorithm however it comes to a less precise solution. Sethanan and Pitakaso (2015) added local search into DE process to solve GAP. They introduced k-variable move that is developed from SWAP algorithm. SWAP algorithm choose only two variables to exchange while k-variable move allows k variables to exchange. DE is also used in many problems such as transportation scheduling problem (Sethanan & Pitakaso, 2016), Generalized Assignment Problem (Sethanan & Pitakaso, 2015), location-allocation problem (Thongdee & Pitakaso, 2015), location problem (Mayacheerw & Pitakaso, 2012) etc.

### Case study

The transportation of live animal has to be a special issue because of healthy of animals. The time for transportation should be at night to avoiding the damage to the animal from the sunlight. The period of transportation should not be more than 8 hours for healthy of animals. This study has 4 factors to assign. 1. Types of truck, which is used in transportation, 2. Hen's farms, which produce 20-week hens, 3. Egg's farms, that need hens to produce eggs, and 4. Round of transportation, if it is needed.

In recent time, hen transportation is not systematic and do not have well-management. The used truck is easy to pick. The hen's farm or the egg's farm are assigned by using minimum distance. Truck, hen's farm, and egg's farm should be assigned simultaneously because the cost of assignment depends on these 3 factors. The distance between hen's farm and egg's farm show transportation cost and travel time. Every factor has an effect on assignment cost.

The flow of transportation starts from the depot. The selected truck go to hen's farm to load hens up then deliver them to egg's farm. After load hens down, this truck will return to the depot. Each truck is cleaned and sterilized then it ready for next transportation. There are 4 types of truck which are ten-wheel truck, six-wheel truck, modified four-wheel truck, and ordinary four-wheel truck. Each type of the truck has different capacity, fuel consumption, and maintenance costs. There are some restrictions for trucks; 1) each type of truck have their working hour. Ten-wheel truck, six-wheel truck, modified four-wheel truck and four wheel truck

have 56, 72, 80, and 112 hours, respectively. 2) Transportation time which includes all loading time is less than 8 hours and should be at night.

Hen's farm produce 20 weeks old hens and export them to egg's farm. Egg's farm import hens from hen's farm and prepare them ready for egg production. This study is a connection these two type of farm. The problem is how to assign hen's farm and egg's farm together with types of truck. Once assign these farms, the distance between two farms is major assignment cost. The amount of hens and capacity of truck may not exactly match. Some truck may have available space that should be full capacity. To avoid available space, the opportunity cost is applied as another assignment cost. Therefore, assignment costs have two parts which are transportation cost and opportunity cost.

The assigned quantity is chosen from the minimum amount that compares between demand from egg's farm and supply of hen's farm. For example, if hen's farm supplies of 4,000 hens and egg's farm demands of 3,000 hens, then the assigned quantity should be 3,000 hens. According to assigned quantity, type of truck is chosen.

This assignment problem is not one to one basis. Numbers of hen's farm is less than numbers of egg's farm. There are some restrictions such as 1) hen's farm can export all of their hens. Hence, after last assignment, no hen left behind (supply is zero). According to a larger numbers of egg's farm, some egg's farms may not meet their full demand. 2) All egg's farms get at least half of their demand. Egg's farms are assigned half of their demand at the beginning. When last egg's farm gets half of demand, second half of demand will be assigned. Consequently, some last assigned farms may not have full of their demand.

### Mathematical model and restrictions

Indices

- $i = 1, 2, 3, 4$  (Types of truck)  
 $j = 1, 2, 3, \dots, J$  (Hen's farm  $J=40$ )  
 $k = 1, 2, 3, 4$  (Round of transportation max  $k = 4$ )  
 $l = 1, 2, 3, \dots, L$  (Egg's farm  $L=60$ )

Parameters

- $c_{ijl}$  is transportation cost of truck type  $i$  for transportation from hen's farm  $j$  and deliver to egg's farm  $l$ .  
 $t_i$  is capacity of truck type  $i$  (unit)  
 $oc_{ijl}$  is opportunity cost when truck type  $i$  for transportation from hen's farm  $j$  and deliver to egg's farm  $l$  does not transport with full load.  
 $d_l$  is quantity demanded of egg's farm  $l$

- $s_j$  is quantity supplied of hen's farm j  
 $tup_{ij}$  is time for lift chicken up to truck i at hen's farm j  
 $zup_{ij}$  is worker's ability of hen's farm j to lift chicken up to truck i  
 $tdn_{il}$  is time for lift chicken down from truck i at egg's farm l  
 $zdn_{il}$  is worker's ability of egg's farm l to lift chicken down from truck i  
 $ttr_{ijkl}$  is transportation time starting from service center to hen's farm j, hen's farm j to egg's farm l, and egg's farm l back to service center (when  $x_{ijkl} = 1$ )  
 $td_{jl}$  is transportation time from hen's farm j to egg's farm l. (for all combination)  
 $time_{ijkl} = tup_{ijkl} + ttr_{ijkl} + tdn_{ijkl}$   
 $ttruck_i$  is service time of truck type i

Decision variables

- $x_{ijkl}$  is decision variable  
 If  $x_{ijkl} = 1$  when truck type i for transportation chicken from hen's farm j in round k and deliver to egg's farm l.  
 If  $x_{ijkl} = 0$ , otherwise.  
 $q_{ijkl}$  is number of chicken that are picked up by truck i for transportation from hen's farm j in round k and deliver to egg's farm l.

Objective function

$$\min \sum_i^I \sum_j^J \sum_k^K \sum_l^L (c_{ijl} \times x_{ijkl}) + \sum_i^I \sum_j^J \sum_k^K \sum_l^L \left[ \left( (x_{ijkl} \times t_i) - q_{ijkl} \right) \times oc_{ijl} \right] \quad \text{eq. 1}$$

Subject to

$$x_{ijkl} \in \{0,1\} \quad \text{for } \forall i j k l \quad \text{eq. 2}$$

$$q_{ijkl} \in 0 \& I^+ \quad \text{for } \forall i j k l \quad \text{eq. 3}$$

$$\sum_l^L x_{ijkl} \leq 1 \quad \text{for } \forall i j k \quad \text{eq. 4}$$

$$\sum_i^I \sum_j^J \sum_k^K q_{ijkl} \leq d_l \quad \text{for } \forall l \quad \text{eq. 5}$$

$$\sum_i^I \sum_j^J \sum_k^K q_{ijkl} \geq 0.5 \times d_l \quad \text{for } \forall l \quad \text{eq. 6}$$

$$\sum_i^I \sum_k^K \sum_l^L q_{ijkl} = s_j \quad \text{for } \forall j \quad \text{eq. 7}$$

$$q_{ijkl} \leq M \times x_{ijkl} \quad \text{for } \forall i j k l \quad \text{eq. 8}$$



$$q_{ijkl} \leq t_i \quad \text{for } \forall i j k l \quad \text{eq. 9}$$

$$tup_{ij} = zup_{ij} \times q_{ijkl} \quad \text{for } \forall i j k l \quad \text{eq. 10}$$

$$tdn_{il} = zdn_{il} \times q_{ijkl} \quad \text{for } \forall i j k l \quad \text{eq. 11}$$

$$ttr_{ijkl} = td_{jl} \times x_{ijkl} \quad \text{for } \forall i j k l \quad \text{eq. 12}$$

$$time_{ijkl} = tup_{ijkl} + tdn_{ijkl} + ttr_{ijkl} \quad \text{for } \forall i j k l \quad \text{eq. 13}$$

$$time_{ijkl} \leq 8 \quad \text{for } \forall i j k l \quad \text{eq. 14}$$

$$\sum_j^J \sum_k^K \sum_l^L time_{ijkl} \leq ttruck(i) \quad \text{for } \forall i \quad \text{eq. 15}$$

$$x_{ijkl} \geq x_{ij(k+1)l} \quad \text{for } \forall i j k l \quad \text{eq. 16}$$

The objective function of this study has two parts. 1) In one assignment, truck is assigned for transportation from hen's farm to egg's farm, therefore, assignment cost or transportation cost exists. 2) When there is available space in transportation truck this is an opportunity cost. Thus the objective of this study is to minimize these two costs together.

There are some restrictions on this model which are 1)  $x_{ijkl}$  is decision variable that must be 0 or 1 (eq. 2). 2)  $q_{ijkl}$  is another decision variable that is quantity of hen to transport for each assignment and it have to be zero or positive integer (eq. 3). 3) Egg's farm can receive hen from only one hen's farm or nothing each time (eq. 4). 4) Each egg's farm may not get full of demand (eq. 5) 5) Egg's farm will get at least 50% of its demand (eq. 6). 6) Hen's farm must export all of their supply (eq. 7) 7) When i, j, k & l are not assigned ( $x_{ijkl} = 0$ ), assigned quantity must be zero (eq. 8) 8. Assigned quantity is less than truck capacity (eq. 9) 9) Lift up time depends on ability of worker from hen's farm and assigned quantity (eq. 10) 10) Lift down time depends on ability of worker from egg's farm and assigned quantity (eq. 11) 11) Transportation time starts from service center to hen's farm, hen's farm to egg's farm and egg's farm back to service center (eq. 12) 12) Total time is lift up time, transportation time, and lift down time (eq. 13) 13) Total time that is used in one assignment is less than or equal 8 hours (eq. 14) 14) Total time of each type of truck is less than service time of that type (eq. 15) 15) Round one is assigned first then round two exists (eq. 16).

### Proposed Heuristics

When an exact method takes a long time to compute the best solution, heuristics method spends very short time to solve the same problem however an answer is acceptable. According to time spending and acceptable answer, the heuristic algorithm should be used.

There are many heuristic algorithms that are common. DE is one of the most popular heuristics. Any heuristics starts with creating initial population, encoding and decoding method which are important. After initial population is set, DE repeatedly does mutation, recombination, and selection until end condition satisfy.

**Table 1:** Distance (unit: km)

Hen's farm	Egg's farm			
	1	2	3	4
1	344	272	256	335
2	293	260	305	381
3	330	259	254	340
4	310	276	229	225

Note: This is distance starting from service center to hen's farm transport to egg's farm then back to service center.

The distances in table 1 start from service center to hen's farm, then transport from hen's farm to egg's farm, and back from egg's farm to service center. The distances are shown in kilometers. The distances are the most important factor that reflects transportation cost. Shorter distance has lower cost while longer distance has a higher cost. Thus the assignment cost depends on the selected distance.

## Encoding

We start with define random number between 0 and 1 for every position of every vector (table 2). Next, ascending sort random number in each vector. The minimum random number will be assigned first and the maximum random number will be assigned last (table 3). We assume 4 positions and 4 vectors for truck's vector, as same as hen's vector and egg's vector.

The first vector of truck's vector, hen's vector, and egg's vector will be used in encoding step. The first truck's vector random numbers are 0.366, 0.374, 0.627, and 0.608 then the rank of each position is 1, 2, 4 and 3 respectively. The random numbers for the first hen's vector are 0.260, 0.318, 0.123, and 0.958 thus the ranks are 2, 3, 1, and 4 respectively. The random number of the first egg's vector are 0.047, 0.394, 0.931, and 0.682, therefore, the ranks are 1, 2, 4, and 3 respectively. All of the encoding ranks are used in decoding step.

**Table 2:** Encoding random number

Position	Truck's vector				Hen's vector				Egg's vector			
	1	2	3	4	1	2	3	4	1	2	3	4
1	0.366	0.033	0.866	0.531	0.260	0.147	0.842	0.404	0.047	0.626	0.837	0.849
2	0.374	0.270	0.546	0.938	0.318	0.295	0.453	0.848	0.394	0.751	0.108	0.582
3	0.627	0.628	0.509	0.093	0.123	0.100	0.347	0.322	0.931	0.205	0.185	0.475
4	0.608	0.746	0.069	0.877	0.958	0.212	0.571	0.803	0.683	0.583	0.274	0.075

**Table 3:** Encoding rank

Position	Truck's vector				Hen's vector				Egg's vector			
	1	2	3	4	1	2	3	4	1	2	3	4
1	1	1	4	2	2	2	4	2	1	3	4	4
2	2	2	3	4	3	4	2	4	2	4	1	3
3	4	3	2	1	1	1	1	1	4	1	2	2
4	3	4	1	3	4	3	3	3	3	2	3	1

Each type of truck has different capacity. Truck type 1 is ten-wheel truck and it has the maximum capacity at 10,000 chicken per transportation. While type 2 (8-wheel truck), type 3 (modified 4-wheel truck), and type 4 (4-wheel truck) can transport 8,000, 4,000, and 2,000 respectively. Hen's farm number 1, 2, 3, and 4 has quantity supplied 10,000, 5,000, 10,000, and 5,000 respectively. Egg's farm number 1, 2, 3, and 4 has quantity demanded 10,000, 8,000, 10,000, and 5,000 respectively. Next, we match truck type rank with its capacity. Then we match quantity supplied of each hen's farm to its rank. Last we match quantity demanded of each egg's farm with its rank. All of the matchings are shown in table 4.

**Table 4:** Rank of positions, the capacity of truck, quantity demanded and quantity supplied

Types of truck		Hen's farm		Egg's farm	
Type	Capacity	Farm number	Quantity supplied	Farm number	Quantity demanded
1	10,000	2	5,000	1	10,000
2	8,000	3	10,000	2	8,000
4	2,000	1	10,000	4	5,000
3	4,000	4	5,000	3	10,000

## Decoding

The first stage of assignment is assign hen's farm to egg's farm. The first hen's farm (#2) has quantity supplied 5,000. The first egg's farm (#1) has quantity demanded 10,000. Assigned quantity is assigned by using minimum amount between quantity demanded and quantity supplied, therefore the assigned quantity is 5,000. Next stage assign type of truck that match the assigned quantity. The first type of truck in queue is 10-wheel truck which has capacity at 10,000. If capacity of truck is 10,000 and assigned quantity is 5,000 then the available space is 5,000. On alternative choice, if the second type of truck is selected, empty space is only 3,000. The rest types of truck are not chosen because their capacity is lower than assign quantity. If they are selected, there are more transportation cost of second round or more. Distance from this assignment is 293 kilometers. The transportation cost is 1,758 baht (5.0 km/liter and 30 baht/liter) and the opportunity cost is 659.25 baht (remains 3,000 over capacity 8,000) thus the assignment cost is 2,417.25 baht. Time use for this type of truck is 3.67 hours (80 km/hour).

After above assignment, quantity demanded of egg's farm number 2 remains 5,000 while hen's farm number 1 is empty. The second hen's farm (#3) has quantity supplied 10,000. The next assigned quantity is 5,000. Truck type 2 recalls again. Distance from hen's farm 3 to egg's farm 1 is 330 km. The transportation cost is 1,980 baht and the opportunity cost is 742.5 baht thus the assignment cost is 2,722.50 baht. Time use of truck type 2 is 4.13 hours. Total time of truck type 2 is 7.8 hour.

Decoding process start by assign hen's farm and egg's farm then we know assigned quantity and distance between those farms. Next, we assign type of truck that match assigned quantity. Transportation cost and opportunity cost are computed. Then, time use of selected truck type is recorded. The loops are repeated until meet end condition.

Like any algorithm, DE starts with initial population or the first generation then follow its algorithm. Then, mutation process which is combination of 3 vectors that will expand the search space. Mutation process starts by randomly select three vectors ( $x_{r1,G}$ ,  $x_{r1,G}$  and  $x_{r1,G}$ ) then uses the weighted difference between two vectors ( $F(x_{r2,G} - x_{r3,G})$ ) and add to the last vector. This vector is called the donor vector.

$$v_{i,G+1} = x_{r1,G} + F(x_{r2,G} - x_{r3,G})$$

Where F is the mutation factor which varies from zero to two.

$x_{r1,G}$ ,  $x_{r1,G}$  and  $x_{r1,G}$  are randomly selected from the target vector.

$v_{i,G+1}$  is called the donor vector or mutant vector.

Recombination step will choose the target vector ( $x_{i,G}$ ) or the donor vector ( $v_{i,G+1}$ ) by using specify probability (CR). If CR is greater than or equal random probability, then the donor vector will be chosen. On the other way, if CR is less than random probability, then the target vector will be selected. Normally, CR is around 0.8 because the donor vector is preferred.

$$u_{j,i,G+1} = \begin{cases} v_{i,G+1} & \text{if } rand_{ji} \leq CR \text{ or } j = I_{rand} \\ x_{j,i,G} & \text{if } I_{rand} > CR \text{ or } j \neq I_{rand} \end{cases}$$

Where  $rand_{j,i}$  is random number between 0 and 1.  
 $I_{rand}$  is a random integer from 1 to D and D is number of dimensions.  
 $I_{rand}$  guarantees that  $v_{i,G+1} \neq x_{i,G}$ .

The fitness function is not mentioned in mutation and recombination. It is used in selection. In DE, selection will select the best fitness value from the trial vector ( $u_{i,G+1}$ ) or from the target vector ( $x_{i,G}$ ). The best vector that has the best fitness value will be the target vector of next generation.

$$x_{i,G+1} = \begin{cases} u_{i,G+1} & \text{if } f(u_{i,G+1}) \leq f(x_{i,G}) \\ x_{i,G} & \text{otherwise} \end{cases}$$

Mutation, recombination, and selection continue until an end condition is satisfied. It might be a number of iterations or time limit.

### Computational framework and results

We begin with testing the mathematical model by using LINGO as exact method and Differential Evolution algorithm (DE) as heuristic method. We generate 3 groups, classified by size of sample. Types of truck are fixed at 4 different types which are 10-wheel truck, 8-wheel truck, modified 4-wheel truck and ordinary 4-wheel truck. Normally, we should transport chicken only one round but we set the maximum round at 4 rounds. Thus size of sample depends on a number of hen's farm and egg's farm. The 3 groups are 4x4, 10x10, and 20x20. The sizes of sample are classified by time-consuming to solve for the best answer by LINGO. Each group has 5 observations that will have 5 best answers. We will use index to compare the results. If index is 100, it means DE result equal LINGO result. Generally, DE results should have higher value than LINGO or index should be higher than 100.

$$index = \frac{DE \text{ result}}{LINGO \text{ result}} \times 100$$

**Table 5:** Results for small size (4x4)

Set	LINGO	DE				Index
		average	max	min	sd	
set 1	9,723	11,799	13,442	9,723	1,374.4	121.4
set 2	8,753	10,348	12,233	8,753	1,317.7	118.2
set 3	5,330	6,416	7,019	5,330	701.7	120.4
set 4	7,056	8,420	9,981	7,056	1,093.4	119.3
set 5	7,317	8,252	9,218	7,317	702.8	112.8

Even though an index show that DE answers have higher results than LINGO's answers. One of DE result of each set has the same answer as LINGO's answer. Time consuming of LINGO in small size is around 6 seconds. (table 5)

**Table 6:** Result for medium size (10x10)

Set	LINGO	time	DE	time	Index
Set 1	11,989.8	175	12,990.4	0.6	108.3
Set 2	11,397.7	79	12,136.0	0.6	106.5
Set 3	12,613.3	93	13,364.2	0.6	106.0
Set 4	10,902.1	92	11,851.7	0.6	108.7
Set 5	12,114.7	79	12,734.4	0.7	105.1
<b>average</b>	<b>11,803.5</b>	<b>103.6</b>	<b>12,615.3</b>	<b>0.61</b>	<b>106.9</b>

From table 6, LINGO spends about 2 minutes to find an answer while DE uses time less than 1 second (in average). In medium size, an index shows that DE more close to LINGO than small size. An average index is 106.9. It is worth to use DE to find the solution than LINGO because of less time consumption.

**Table 7:** Result for large size (20x20)

Set	LINGO	DE	Time	Index
Set 1	30,139.0	32,716.9	5.1	108.55
Set 2	26,868.0	29,094.6	5.1	108.29
Set 3	33,688.0	36,128.6	3.6	107.24
Set 4	27,097.0	28,049.1	3.7	103.51
Set 5	27,017.0	29,152.5	3.7	107.90
<b>average</b>	<b>28,961.8</b>	<b>31,028.3</b>	<b>4.2</b>	<b>107.10</b>

From table 7, we spend 72 hours for LINGO to find an answer but it can find only feasible answer. Thus, it is valuable to use DE instead of LINGO because DE spends only 4

seconds (in average). Indices of medium and large sample are almost the same. An index shows that DE's result is slightly higher than LINGO's result however time consumption is steeply lower.

**Table 8:** Result for case study

	DE	time	Index
de 1	122,461.9	6.4	106.6
de 2	116,979.8	10.3	101.8
de 3	127,749.6	8.7	111.2
de 4	129,933.9	7.8	113.1
de 5	129,150.9	6.0	112.4
<b>average</b>	<b>125,255.2</b>	<b>7.8</b>	<b>109.0</b>

From table 8, we run LINGO for 72 hours for case study then it shows only feasible solution. The fitness function is 114,932. The best answer should be lower than this number. We run DE for 5 times then it has an average at 125,255 with index at 109.0. DE's answer is slightly higher than LINGO answer however DE running time is steeply shorter than LINGO running time.

## Conclusions

The problem in this paper is Multi-Stage Assignment Problem. There are two ways to solve this problem. The first one uses mathematical model and LINGO to solve for the exact solution. When there is large size of problem or complicated problem, it takes very long time to solve. Differential Evolution (DE) is an alternative method to find an appropriate answer with shorter time. DE will do mutation, recombination and selection, repeatedly. The pros of DE are the expansion of searching area, few parameters, and compute fitness function only in selection process while the cons of DE are the shift of possible best answer area to another area, and it is not perfect for complicated problem.

The exact method (LINGO) show only feasible answer within 72 hours. When we add more restrictions, time consumption will take longer time than usual. Time consumption of DE will be no differentness, significantly. The order from egg's farm is in daily process thus we have to send chicken quickly. We cannot wait 3 days for the lowest cost from LINGO programming. The appropriated cost from DE spends shorter time therefore DE is reasonable to use.

From a case study, if use LINGO to find the best answer, it takes more than 72 hours to find only feasible answer. DE can find an appropriate answer (index = 109) and take around 8 seconds to find this answer. In real business, it is worth to use DE to find how to make an assignment with acceptable answer. No need to find the best answer by using LINGO and wait for more than 3 days. The result from DE might be slightly different from the best answer however steeply low time consumption.

To improve DE result, modified DE should be developed thus modified DE result might be closer to the best answer. The modified DE direction is how to make DE stick with possible best answer area and do not expand to another area like in ordinary mutation process. According to DE might not perfect for complicated problem, another heuristic method should be concerned. The others popular heuristics, are Particle Swarm Optimization (PSO), Ant Colony System (ACO) and etc., should be used instead of DE. One of our assumption is "other related factors is fixed". If every factors have only marginal changes, we cannot investigate our research with limited time. If we look at this problem in different way, we might have a better way to solve a problem. We define this is assignment problem. It will be different when we define it as Vehicle Routing Problem (VRP). In the future research, it should have a modified DE or the other heuristic method to solve this case study. The researcher might redefine this assignment problem into a new problem.

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