

An Examination of Common Errors in Geometric Concepts among Senior Secondary School Students in Lagos State

Olujide Dixon, Olabisi Onabanjo University Ago-Iwoye. Email: olujidedixon@gmail.com.

Folashade Ajayi. Email: ogundiran.folashade9@gmail.com.

Abstract

This study examined students' performance in geometry and the specific errors related to their understanding of certain mathematical concepts. The emphasis was on five geometry subjects: constructions, angles, circles, volume, and polygon properties. A quantitative research design was utilised, and data were gathered through the Mathematics Achievement Test (MAT). Students' answers were scored in a systematic way, and an error analysis was done using a structured coding scheme to put mistakes into the following groups: conceptual, procedural, symbolic, reasoning, and careless. Descriptive statistical methods were employed to examine overall achievement, performance across geometric concepts, and the distribution of error types. The results showed that the students' overall mathematics achievement in geometry concept was at a moderate level. But the results were very different for the different geometry concepts that were tested. Students did better on volume and properties of polygons. The analysis of the errors showed that most of the mistakes were conceptual and procedural, which means that many students relied on memorised steps without really understanding what they were doing. Mistakes in reasoning were especially clear in topics that were more abstract and required more thought, like constructions and tasks related to angles. The results show that students' understanding of geometry is not developing evenly, and they stress the need for teaching methods that help students understand concepts, reason, and visualise space. The research emphasises the significance of employing diagnostic assessments to pinpoint particular error patterns and guide focused instructional interventions. The results add to what is already known about how students learn geometry by showing how well they do and what kinds of mistakes they make. This has useful implications for teaching mathematics, developing curricula, and testing methods.

Keywords: error analysis, geometry, geometry concept, mathematics, mathematics achievement

1. Introduction

Errors in mathematics are not treated merely as wrong; they show how students think, how teachers respond to them, and how mathematics information is organised. Arguably, many studies focusses vary from probability and mathematics to trigonometry and word problems, they agree that mistakes are methodical, based on how the brain works, and important for teaching, not just random mistakes (Radatz, 1979; Borasi, 1987; Veloo, Krishnasamy & Wan Abdullah, 2015).

Geometry is an important part of high school mathematics as it helps students learn how to think logically, see things in three dimensions, and solve problems, which are all abilities they need for more advanced science and technology classes. At the Senior Secondary School (SSS) level, geometry encompasses not only basic shape recognition but also the relationships among angles, the attributes of planar and solid objects, coordinate geometry, and trigonometry. Even though geometry is a major part of mathematics, students have always performed less in it than in other subjects. Reports from the classroom assessment often point out mistakes and misunderstandings that keep happening. These persistent difficulties indicate that students' issues in geometry are not solely computational but fundamentally conceptual and representational in essence.

Research in mathematics education has long demonstrated that students' errors yield significant insights into their cognitive processes. Radatz (1979) characterised mathematical errors as systematic results of cognitive, linguistic, and instructional variables rather than arbitrary errors, a perspective that has influenced subsequent research on error analysis. Borasi (1987) posited that errors ought to be regarded as chances for learning, as they elucidate the structure and constraints of students' comprehension. In geometry, where diagrams, symbols, and spatial relationships are typically used to convey meaning, mistakes can show holes in comprehension that are otherwise disguised by memorised steps.

Empirical investigations have shown that errors in geometry often happen when people misread diagrams, use symbols incorrectly, or apply theorems incorrectly. Veloo *et al.* (2015) found representational and problem-solving errors as predominant patterns, whereas Molina *et al.* (2017) demonstrated that students encounter difficulties in converting verbal descriptions into symbolic or graphical representations. Arhin and Hokor (2021) discovered that trigonometric errors among secondary school pupils were significantly associated with inadequate conceptual comprehension and erroneous pictorial interpretation. These results indicate that geometry necessitates a cohesive comprehension of concepts, representations, and reasoning methodologies.

In Nigeria, research by Nwoke *et al.* (2024) and Aguele, Omo-Ojugo, and Imhanlahimi (2010) has documented ongoing conceptual and procedural errors among senior secondary school pupils, highlighting systemic instructional difficulties. Nonetheless, a significant portion of the current literature discusses mathematics in general, with little emphasis on geometry as a specific area of challenge. Moreover, teacher-centric research (Durkaya *et al.*, 2011; Sapire *et al.*, 2016; He, 2025) underscores that teachers' interpretations and reactions to students' errors profoundly affect the correction or reinforcement of misconceptions.

A prominent discourse in the literature is the repudiation of errors as just random occurrences and the acknowledgement of their systematic, interpretable characteristics. Some writers focus on cognitive structures, while others focus on instructional responses, representations, or remediation (Schoenfeld, 2013; Gierl, Alves & Majeau, 2010; Sia & Lim, 2018). These viewpoints enhance the concept of mathematical error, framing it as a complex phenomenon essential for comprehending learning, teaching, and student mathematical accomplishment.

However, reports from various examiner reports in recent years suggest ongoing poor performance of senior secondary school students in geometry is still a big problem in mathematics education, especially in Lagos State, where external exam results show that many students still have trouble grasping geometric ideas. Geometry is an important part of the secondary school mathematics curriculum because it helps students learn how to think logically, see things in three dimensions, and do better in others aspect of mathematics like algebra, calculus, trigonometry among others.

Even though it is important, students still make mistakes in things like angle relationships, attributes of forms, congruence and resemblance, coordinate geometry, and geometric proofs. These persistent errors indicate not mere carelessness but profound conceptual and procedural misconceptions that hinder students' capacity to apply geometric information effectively.

The problem is exacerbated by the high-stakes character of senior secondary examinations, such as the West African Examination Council (WAEC) and the National Examination Council (NECO), in which geometry-related problems constitute a significant amount of the marks. Therefore, pupils who keep making mistakes in geometry do poorly in mathematics generally and could miss out on chances to study more in science, technology, engineering, and mathematics-related disciplines.

In Lagos State, the problem is made worse by the fact that there are many different types of schools and students come from many different backgrounds. This is because the quality of instruction, the expertise of the teachers, and the availability of learning tools all vary greatly. In classrooms nowadays, students are generally taught to memorise formulas and processes instead of understanding the concepts behind them. This makes them more likely to make mistakes when problems are presented in new ways.

Consequently, the problem for this study arises from the necessity to transcend broad assertions of inadequate performance in geometry and instead conduct a comprehensive analysis of the specific errors made by pupils and the fundamental elements that contribute to these mistakes. If we do not know what these mistakes are and where they come from, any teaching methods we try will be guesswork and won't work. Therefore, it is imperative to discern prevalent error patterns in geometric concepts among senior secondary school students in Lagos State and to examine whether these errors arise from misconceptions, flawed reasoning, representational challenges, or the improper application of rules.

The importance of this study is that it could help teachers, students, and policymakers make better decisions about mathematics education. The study will offer actual evidence regarding prevalent error

types in geometry, enabling educators to identify students' challenges and formulate tailored instructional techniques that rectify misconceptions rather than only addressing symptoms. The findings can indicate to curriculum developers and examination bodies content areas that necessitate increased focus or different teaching strategies. The study will augment the existing literature on error analysis in mathematics education, providing context-specific insights crucial for enhancing students' conceptual understanding and performance in geometry in Lagos State and analogous educational environments.

In light of this context, the current study investigates prevalent misconceptions in geometric concepts among senior secondary school students in Lagos State. The study aims to enhance the comprehension of students' geometric reasoning by systematically examining the characteristics and trends of these errors, thereby offering evidence-based insights to improve the teaching and learning of geometry.

1.1. Research Question

What types of errors do senior secondary school students commit in selected geometric concepts (construction, angles, circles, volume, and properties of polygons) in Lagos State?

2. Literature Review

2.1. Types of Errors in Geometry

In the literature, geometry-related errors are uniformly seen as systematic expressions of learners' interpretations of spatial relationships, symbols, and representations, rather than as mere computing errors. Even though studies have different areas of study, methods, and levels of analysis, unarguably, most studies agree that geometric errors happen when conceptual comprehension, representational competence, and instructional mediation work together. Table 1 shows a summary of the numerous types of geometry errors made from different studies.

A key point of agreement is that conceptual errors are very common in geometry. Radatz's (1979) basic paradigm contextualises these errors within expansive cognitive origins, encompassing erroneous associations and inflexible rule application. This elucidates why students could erroneously believe that all triangles possess identical qualities or utilise angle theorems without considering the conditions inherent in diagrams. This conceptual perspective is supported by the works of Chamundeswari (2014), Sarwadi and Shahrill (2014), and Arhin and Hokor (2021), who tacitly see geometric faults as indicators of misconstrued relationships rather than procedural deficiencies. What sets Radatz apart is that he focusses on where mistakes come from, while later writers generally focus on how they show up in specific areas like trigonometry or coordinate geometry.

Rule-application and procedural errors are quite similar to conceptual errors, although different writers have different ideas about how important they are. Dawkins (2006) and Aguele *et al.* (2010) note that people often employ formulas and algorithms in geometry incorrectly, including using the wrong area calculations or building procedures. Aguele *et al.* (2010) argue beyond just listing these mistakes by calling them process defects that can be fixed with specific teaching methods. This is different from Dawkins (2006) more practical view, which sees mistakes as common traps to avoid. The analytical differentiation pertains to the perception of procedural faults as superficial concerns or as manifestations of underlying conceptual vulnerability, a tension that recurs throughout the literature.

Another important area is representational and diagrammatic mistakes, which are especially important in geometry. Veloo *et al.* (2015) distinctly categorise errors resulting from the misreading of symbols, graphs, and issue contexts, a classification that closely corresponds with geometric learning, where meaning is visually conveyed. Molina *et al.* (2017) enhance this viewpoint by framing errors as deficiencies in the translation between representations, including verbal descriptions and algebraic or diagrammatic formats. In geometry, this elucidates why pupils may accurately remember definitions while misinterpreting representations or inadequately correlating textual information with spatial arrangements. Unlike Veloo *et al.* (2015), who categorise representation errors as a distinct group, Molina *et al.* underscore the semiotic intricacy of mathematics, proposing that geometric errors originate from linguistic and symbolic mediation rather than solely from spatial reasoning.

Cognitive coherence provides an additional perspective for comprehending geometric faults. McGowen and Davis (2019) do not explicitly study geometry; rather, their concept-map methodology has significant consequences for geometric comprehension. Their research indicates that errors occur when pupils have disjointed conceptual networks, a situation that is particularly problematic in geometry, since attributes and interactions are interconnected hierarchically. This viewpoint differentiates itself by redirecting focus from evident errors to the foundational structure of knowledge, suggesting that enduring geometric fallacies signify systematic disconnections rather than singular misunderstandings.

Teacher-oriented studies broaden the concept of error beyond student cognition. Durkaya *et al.* (2011), Sapire *et al.* (2016), and He (2025) characterise errors as educational entities whose significance is contingent upon teachers' interpretations and responses. In geometry, where diagrams and spatial reasoning frequently obscure problems, these authors contend that teachers' anticipatory and diagnostic methodologies dictate whether errors transform into educational opportunities or persistent misunderstandings. The emphasis on representational reasoning, while grounded in probability, is analytically applicable to geometry, given that both fields significantly depend on organised diagrams. The unique contribution of this research is their reframing of errors as relational events, co-constructed through classroom interaction rather than merely generated by learners.

Another common element is the persistence of misinformation. Sarwadi and Shahrill (2014) emphasise that repeated exposure without conceptual remediation results in enduring geometric misconceptions, especially among pupils who are repeating. This perspective aligns with Nwoke *et al.* (2024), who regard error patterns as indicators of systemic instructional deficiencies. Nwoke *et al.* concentrate primarily on success outcomes; nonetheless, their findings tacitly endorse the notion that geometry errors are not arbitrary but rather institutionalised through curriculum and instruction.

Unarguably, most of the authors agree that the "error-as-failure" model is not the only one. Errors in geometry are commonly recognised as significant, systematic, and diagnostically useful. Conceptual misunderstanding, representational difficulty, and improper rule application frequently emerge as primary factors. The authors differ in the level at which they position the problem: some focus on cognitive structure, while others highlight symbolic mediation, procedural execution, or instructional response. These viewpoints indicate that geometric errors in senior secondary students are optimally comprehended as multidimensional phenomena, necessitating investigations that combine cognition, representation, and education instead of discrete classification.

Author(s)	Geometry-Related Types of Errors
Veloo, Krishnasamy & Wan Abdullah (2015)	Symbolic errors in geometric notation, misinterpretation of graphs and diagrams, problem-solving errors in spatial tasks
Radatz (1979)	Conceptual errors in geometric ideas, language-related errors in worded geometry problems, incorrect rule application (e.g., theorems), rigidity in spatial reasoning
Borasi (1987)	Conceptual and reasoning errors in geometric thinking; productive errors arising from flawed geometric conjectures
Dawkins (2006)	Rule-misapplication errors in geometry formulas, procedural errors in angle and shape calculations
He (2025)	Representational errors (tree diagrams as spatial structures), conceptual and structural reasoning errors transferable to geometric diagrams
Nwoke <i>et al.</i> (2024)	Conceptual and procedural errors in geometry topics; errors linked to poor understanding of properties of shapes
Durkaya <i>et al.</i> (2011)	Anticipated geometric misconceptions, procedural errors in constructions and proofs, careless diagram interpretation
Chamundeswari (2014)	Conceptual errors in symbolic manipulation relevant to coordinate and algebraic geometry
McGowen & Davis (2019)	Fragmented conceptual structures affecting geometric relationships and spatial coherence

Author(s)	Geometry-Related Types of Errors
Arhin & Hokor (2021)	Conceptual errors in trigonometry, diagram interpretation errors, incorrect use of geometric relationships
Sarwadi & Shahrill (2014)	Persistent misconceptions in geometric concepts, procedural errors in angle and shape properties
Aguele <i>et al.</i> (2010)	Process errors in geometry problem solving, algorithmic errors in constructions and mensuration
Sapire <i>et al.</i> (2016)	Learners' reasoning errors in geometry, instructional use of geometric misconceptions
Priyani & Ekawati (2018)	Conceptual and procedural errors in geometry-related TIMSS items, reasoning errors in spatial tasks
Molina <i>et al.</i> (2017)	Translation errors between verbal descriptions and algebraic/diagrammatic geometric representations
Haghverdi, Semnani & Seifi (2012)	Linguistic and semantic errors in geometry word problems; strategic errors in spatial problem solving
Abdul Rahman & Mohamad Foad (2021)	Conceptual and graphical errors in quadratic functions linked to coordinate geometry

Table 1: Geometry-Related Types of Errors; source: Authors' Compilation

2.2. Theoretical Perspective

Constructivist theory serves as a fundamental theoretical framework for comprehending and analysing the nature of students' errors in geometric concepts, which is the primary emphasis of this study. Constructivism asserts that learners actively construct mathematical knowledge by engagement with tasks, representations, and prior experiences, rather than passively absorbing information conveyed by the teacher (Scholnik, Kol & Abarbanel, 2006). From this viewpoint, learning constitutes a process of sense-making, with errors being a normal and unavoidable result of students' efforts to integrate new information with pre-existing cognitive frameworks (Piaget, 1978; von Glasersfeld, 1995). In geometry, where ideas like angle connections, congruence, resemblance, and spatial reasoning are quite abstract, students often rely on what they already know and understand, which can lead to systemic misunderstandings.

In a constructivist paradigm, learners' geometric errors are not regarded as mere oversights but as indicators of incompletely formed knowledge. For instance, a student who continuously believes that all triangles with equal sides are right-angled is exhibiting an internally consistent yet erroneous conceptual framework. Borasi (1987) contends that these errors are beneficial as they expose the learner's cognitive processes and create possibilities for conceptual reorganisation. This is exactly what the current study is trying to do: look at the common mistakes senior secondary school pupils in Lagos State make when it comes to geometric concepts in order to learn more about how they think about geometric relationships.

Constructivist approach underscores the significance of representations and social interaction in the learning process. Diagrams, symbols, and words are very important in geometry, and students must actively analyse and connect various representations to make sense of them. Research by Molina *et al.* (2017) and Veloo *et al.* (2015) illustrates that students' challenges in translating between representations frequently lead to enduring geometric inaccuracies. From a constructivist perspective, these errors signify disruptions in meaning-making rather than deficiencies in memory or instruction.

Constructivism also emphasises the teacher's function as a facilitator, directing students to contemplate and enhance their comprehension. Sapire *et al.* (2016) and Durkaya *et al.* (2011) demonstrate that when educators address students' errors diagnostically, they facilitate conceptual transformation rather than just rote correction. In the context of this study, constructivist theory validates the examination of students' blunders as a method to uncover their constructed comprehension of geometry and to guide instructional tactics that enhance profound conceptual learning.

3. Research Methodology

This study utilised a quantitative research methodology, incorporating a Mathematics Achievement Test (MAT) alongside content analysis of students to investigate errors in geometric concepts. The study population consisted of Senior Secondary School II (SSS II) pupils from public secondary schools in Lagos State. The study included a total of three hundred sixty (360) SSS II students, who were chosen from three different school districts in the state.

The sample was chosen using a multi-stage sampling method. In the first stage, three of the six educational districts in Lagos State were chosen using purposive sampling. Educational Districts II, III, and V were selected due to their significant concentration of public senior secondary schools, assuring sufficient representation of students and institutions. In the second stage, nine public senior secondary schools were chosen at random in a way that was proportional to the number of schools in each of the three districts. This method made sure that each district's contribution to the sample was fair.

In the final stage, the study utilised convenience sampling to pick 40 SSS II students from each of the nine schools. This gave us a total sample of 360 students. The participants were selected from entire SSS II classes, as reorganisation of classes was impractical within the educational environment. Before the test, all of the chosen students had learnt about the geometric themes that were on it. Students' script on the MAT was examined to ascertain the errors in geometry assessment.

The Mathematics Achievement Test (MAT) was the instrument used to collect data. It had 20 open-ended essay questions that were meant to test students' comprehension of certain geometric ideas. The same MAT were administered to the participants in the study. There were four questions in the test, one from each of these areas: construction, angles, circles, volume, and properties of polygons. The items were taken from WAEC Mathematics essay questions from 2020 to 2023, making sure they were relevant to the curriculum and valid in terms of substance.

SSS II students were deemed suitable for the study since they constitute a transitional cohort that has encountered the requisite geometric material but has not yet participated in external assessments. This reduced the pressure associated with examinations and facilitated a more genuine evaluation of students' comprehension and error tendencies in geometry.

The instrument used for the data analysis is the Mathematics Achievement Test (MAT) to do the data analysis. All response scripts were marked in a systematic way, and each student's score was written down. There was a total of 100% possible points on the test, with each open-ended essay question worth up to five points. After the tests were graded, the students' answers were closely looked at to find mistakes made on each test item. A structured coding system was used to sort the mistakes into five groups: conceptual errors (1), procedural errors (2), symbolic errors (3), reasoning errors (4), and careless errors (5). After that, descriptive statistical methods were used to summarise how well the students did overall and to find patterns in their scores, as well as the number and spread of each type of mistake across the test items patterns in their scores.

4. Findings

4.1. Demographic Characteristics

The demographic data showed that 173 of the respondents, or 48.1%, were male, while 187, or 51.9%, were female. Also, 145 (40.3%) are between the ages of 16 and 17, 142 (39.4%) are between the ages of 14 and 15, and the last 73 (20.3%) are 17 or older.

4.2. Performance in MAT

This study analysed students' mathematical performance in five geometry concepts: constructions, angles, circles, volume, and properties of polygons. The results show that the average score for mathematics was 52.36%, which is a moderate level. Students performed better in volume, with an average score of 14.21 (SD = 2.45) out of 20 marks. This was the highest level of achievement for any of the geometry concepts tested. This was closely followed with their performance in the properties of polygons (M = 12.67, SD = 2.80) and circles (M = 10.89, SD = 2.67). On the other hand, students'

performance in angles ($M = 9.56$, $SD = 2.29$) and constructions concept ($M = 8.89$, $SD = 2.59$) had the lowest achievement levels, representing the geometry concept where students are weak.

4.3. Types of Errors Committed by SSS II in Selected Geometric Concepts

	Type of Error					Total
	1	2	3	4	5	
Construction						
Construction of angles	4	2	0	0	2	8
Construction of triangles given specified conditions	5	3	3	2	0	13
Construction of perpendicular	6	2	-	5	2	15
Construction of bisectors	5	1	2	2	1	11
Angles						
Angle relationships in parallel lines	4	3	3	2	2	14
Angles in triangles	5	0	0	0	2	7
Angles in polygons	4	2	0	1	4	11
Angles formed by intersecting lines	3	3	0	2	-	8
Circles						
Properties of angles in a circle	3	2	0	2	2	9
Tangents to a circle	2	3	3		2	10
Chords and arcs	2	0	1	3	1	7
Cyclic quadrilaterals	0	2	2	2	0	6
Volume						
Volume of prisms	4	2	1	1	0	8
Volume of pyramids	0	2	0	0	2	4
Volume of spheres	0	0	3	2	1	6
Application of volume formulas to word problems	6	2	0	1	2	11
Properties of Polygons						
Properties of polygons	5	0	3	2	0	10
Sum of interior and exterior angles of polygons		2	1	0	0	3
Properties of special polygons	6	2	2	0	3	13
Diagonals and symmetry in polygon	6	2	0	0	4	12
Total	70	35	24	27	30	186
Total (%)	35.2	17.6	12.1	13.6	15.1	

Table 2: Distribution of Geometry-type Errors Based on the Topics; source: Original Research

Table 2 shows that students made a total of 186 mistakes in their answers. Of these, conceptual errors were the most common, making up 70 errors (35.2%). Then came procedural mistakes, which were observed 35 times (17.6%), and careless mistakes, which happened 30 times (15.1%). There were 27 reasoning errors (13.5%) and 24 symbolic errors (12.1%), which were the least common.

For the measuring concept, most mistakes were made in geometric construction (47) and angles (40), which shows that these were the most difficult topics for students. On the contrary, there were fewer mistakes in circles (32) and volume (29), which means that student's performance was better than the others geometry concept measured.

Further analysis by the researcher showed that most of the conceptual errors were caused by students not understanding how to use construction principles, angle relationships, circle theorems, three-dimensional properties, and the properties of polygons. Most of the procedural mistakes were due to wrong construction steps, wrong use of angle rules, and wrong use of geometric theorems. Symbolic mistakes included writing down arcs and angles wrong and giving angles the wrong name. Careless

mistakes were noticed where students did not convert units correctly or didn't use mathematics tools properly. Reasoning mistakes were observed where participants made invalid deductions or drew wrong logical conclusions while trying to solve a problem.

4.4. Demographic Characteristics with Types of Errors Committed by SSS II in Selected Geometric Concepts

Gender	Type of error					Total
	1	2	3	4	5	
Male	40	18	12	17	16	103
	21.5%	9.7%	6.5%	9.1%	8.6%	55.4%
Female	30	17	12	10	14	83
	16.1%	9.1%	6.5%	5.4%	7.5%	44.6%
Total	70	35	24	27	30	186
	37.6%	18.8%	12.9%	14.5%	16.1%	100.0%

Chi-square=1.27, df=4, p=0.87

Table 3: Gender and Types of Geometry Errors among SSS II Students; source: Original Research

From Table 3, It can be found that a greater number of male students, 40, 21.5% of the total, had made conceptual errors as opposed to 30 female students, 16.1%, an indication that the number of conceptual errors made by male students is greater than that of female students. As to procedural errors, of the male students, 18, 9.7%, made procedural errors, while 17 female students, 9.1%, committed procedural errors, showing that the difference is slight.

A breakdown of errors of different types revealed that male students committed fewer symbolic errors (12, 6.5%), while female students committed fewer reasoning errors (10, 5.4%). Also, both male (21.5%) and female (16.1%) students had the highest number of conceptual errors than procedural, symbolic, and reasoning errors. The chi-square test ($\chi^2 = 1.27$, $p = 0.87$), however, indicates that there is no statistically significant relationship between students' gender and the types of errors committed suggesting gender did not play a significant role in the distribution of errors observed in the geometry assessment.

Age	Type of error					Total
	1	2	3	4	5	
14-15yrs	18	16	6	5	11	56
	9.7%	8.6%	3.2%	2.7%	5.9%	30.1%
16-17yrs	40	15	17	15	15	102
	21.5%	8.1%	9.1%	8.1%	8.1%	54.8%
17+yrs	12	4	1	7	4	28
	6.5%	2.2%	0.5%	3.8%	2.2%	15.1%
Total	70	35	24	27	30	186
	37.6%	18.8%	12.9%	14.5%	16.1%	100.0%

Chi-square=11.93, df=8, p=0.15

Table 4: Age and Types of Geometry Errors among SSS II Students; source: Original Research

Evidence from Table 4 revealed that students within the age of 16 years and 17 years (40 or 21.5%) commit more conceptual errors than those within the age of 14 years and 15 years (18 or 9.7%) and that those that are 17 years (12 or 6.4%). Similarly, students within the age of 16 years and 17 years old (8.1%) recorded more procedural symbolic (17(9.1%), reasoning (15 or 8.1%) and careless (15 or 8.1%) errors in geometry assessment than others age group. A cursory look at the results revealed that students between the age of 16 years and 17 years old recorded more errors in the geometry assessment 102(54.8%). This was followed by students within the age of 14 years and 15 years (56 or 30.1%) while students that are 17 years above 28 or 15.1% recorded the least errors. Furthermore, most of the errors

are committed in the conceptual (70 or 37.6%). This follows by procedural errors (35(18.8%) and careless errors (30(16.1%) respectively. More also, fewer errors were recorded in reasoning 27(14.5%) and symbolic 24(12.9%). However, the chi-square test result ($\chi^2 = 11.93$, $p = 0.15$) indicates that there is no statistically significant relationship between students' age and the type of errors recorded. This suggests that age did not significantly associate with the pattern of errors observed in the geometry assessment.

5. Discussion

The results of this study provide significant insights into students' comprehension of geometry and the characteristics of their mathematical challenges. While the overall level of maths achievement was average, the fact that students did better on some geometry concepts than others shows that their understanding is based on the concepts rather than being evenly developed. This pattern corresponds with constructivist perspectives on learning, which assert that mathematical knowledge is developed progressively and is significantly influenced by the depth of conceptual comprehension rather than mere exposure.

Students' performance on the volume concept shows that they have a relatively good grasp of how to measure space and use formulas. This outcome may be ascribed to the tangible and visual characteristics of volume, which frequently incorporate manipulatives, authentic contexts, and procedural routines that students repeatedly experience during classroom instruction. Earlier research has indicated enhanced performance in measurement-related subjects, positing that concrete representations facilitate conceptual clarity and procedural fluency (Battista, 2007; Clements & Sarama, 2011). However, although high performance in volume indicates procedural proficiency, it does not inherently signify profound conceptual comprehension, as students may depend significantly on memorised formulas without fully understanding the foundational principles. This raises concerns articulated by Skemp (1976) pertaining to instrumental rather than relational comprehension in mathematics education.

The comparatively high proficiency in the properties of polygons and circles indicates that students are more at ease with subjects that focus on definitions, classification, and identifiable visual characteristics. Van Hiele's (1986) research corroborates this interpretation, indicating that numerous students function effectively at lower van Hiele levels, characterised by recognition and description, while encountering difficulties when abstract reasoning or formal deduction is necessitated. This might explain why performance drops in areas like angles and constructions, which require more advanced reasoning, spatial visualisation, and logical sequencing.

The weakest performance, seen in constructions, is very important because construction tasks need to combine conceptual knowledge, procedural accuracy, and spatial reasoning. The frequency of mistakes in this domain corroborates previous studies suggesting that geometric construction is intellectually challenging and frequently undervalued in educational practices (Sinclair & Bruce, 2015). When students don't have chances to use tools like compasses and rulers in a meaningful way, construction tasks become mechanical instead of conceptual. This leads to more procedural and careless mistakes.

Likewise, poor performance in angles indicates challenges in abstract reasoning and the integration of various geometric relationships. Mitchelmore and White's (2000) research shows that students often have broken concept images of angles, seeing them as static shapes instead of ways to measure rotation. This misunderstanding can elucidate both conceptual and reasoning errors identified in the current study, especially when angle problems are presented in unfamiliar orientations.

The error analysis offers additional explanatory depth by demonstrating that students' difficulties stem not solely from insufficient effort, but from inherent cognitive challenges. The prevalence of conceptual and procedural errors across items suggests that numerous students find it challenging to associate mathematical concepts with suitable solution strategies. This finding aligns with Newman (1977) and subsequent research, which posits that errors frequently indicate deficiencies in comprehension rather than mere carelessness. Nonetheless, the occurrence of careless errors indicates that elements such as test anxiety, time constraints, or inadequate self-monitoring abilities may also lead to diminished performance, as observed by Ashcraft and Kirk (2001).

Some studies contradict these findings, indicating better student performance in angle concepts relative to volume, especially in contexts where instruction prioritises visual reasoning and dynamic geometry software (Laborde, 2005). This difference shows that the way teachers teach and the places where students learn are very important for how well they do in geometry. The moderate overall achievement noted in this study may indicate not only the students' capabilities but also deficiencies in instructional strategies that emphasise procedural completion rather than conceptual engagement.

6. Conclusion

The results show that the overall students' performance in geometry concept of mathematics is moderate while their scores on different geometry concepts were very different. This suggests that their understanding of the concepts was not consistent. Better performance was reported in volume, polygon properties, and circles means that students are more comfortable with and familiar with topics that are more concrete or easy to see. On the other hand, poor performance in angles and constructions shows that students still have trouble with abstract reasoning, spatial visualisation, and putting together different geometric ideas. These differences show how important it is to think about the cognitive demands of each geometry topic instead of treating geometry as a single field.

The results also show that the cognitive demands of each concept, the way the teacher teaches, and the level of understanding of the concepts in the classroom all affect how well students perform in geometry. To help students who keep making mistakes, especially when it comes to concepts and reasoning, teachers need to use teaching methods that focus on visualisation, reasoning, and actively building knowledge instead of just following rules.

The examination of error patterns substantiates the assertion that students' difficulties are fundamentally attributed to conceptual and procedural misconceptions rather than mere random errors. The frequency of these errors indicates that numerous students depend on rote memorisation and algorithmic methods without comprehending the fundamental mathematical principles. There were careless mistakes, but they weren't the main reason for low achievement. This means that low achievement can't be blamed only on not paying attention or not trying hard enough. Instead, the results show that there are holes in how teachers teach that may not put enough emphasis on reasoning, clear concepts, and meaningful engagement with geometric ideas.

These findings have significant impact on classroom teaching; curricula design and assessing students in mathematics. Teachers' need to prioritise the enhancement of students' conceptual understanding through exploratory activities, visual representations, and experiential learning, especially in complex concept such as angles and constructions. Teaching methods that could assist students to improve their thinking, justify procedures, and reflect on errors may help reduce conceptual and reasoning errors should be employed. Also, using dynamic geometry tools and manipulatives can help students think spatially and better understand abstract geometric relationships.

The results indicate that the curriculum requires a more balanced methodology that combines procedural fluency with conceptual comprehension in all geometry subjects. Mathematics classroom assessment should not only check for correct answers, but also look for certain kinds of mistakes so that teachers can give students specific help. Finally, teacher professional development programs should focus on improving teachers' knowledge of how to teach geometry, especially when it comes to teaching difficult and abstract ideas. This study underscores the necessity for instructional methodologies that transcend mere procedural proficiency, aiming for a more profound and cohesive comprehension of mathematics.

6.1. Limitations and Areas for Further Research

This study produced valuable insights; however, several limitations must be recognised. First, the study only used data from the Mathematics Achievement Test (MAT), which is good for judging how well students did and what mistakes they made, but it might not fully show how well they understood concepts or how they reasoned. Written responses alone restrict the capacity to examine students' cognitive processes, misconceptions, and decision-making strategies more comprehensively. The

research concentrated on a narrow range of geometry concepts, potentially limiting the applicability of the results to other mathematical concepts.

Second, the sample size and the context in which it was taken may also make it difficult to apply the results to other situations. If the participants were selected from a particular school, region, or educational environment, the results may indicate contextual influences such as instructional methodologies, curriculum focus, or teacher proficiency, rather than overarching trends in mathematics education. Moreover, the study's cross-sectional design offers merely a momentary view of students' performance, failing to consider the evolution of comprehension or error trends over time.

Future research should rectify these limitations by employing mixed-methods approaches that integrate quantitative evaluations with qualitative data, including interviews and classroom observations. These methods would give a better understanding of how students think and why they make different kinds of mistakes. Longitudinal studies may also be performed to investigate the progression of students' geometric comprehension and error patterns across different grade levels. Moreover, subsequent research should investigate the effects of particular instructional interventions on diminishing conceptual and reasoning errors.

7. References

Abdul Rahman, T.F. & Mohamad Foad, M.S. (2021). Quadratic functions in additional mathematics and mathematics: An analysis on students' errors. *Academic Journal of Business and Social Sciences*, 5(1), 1-16, available at: <https://ir.uitm.edu.my/id/eprint/59114/>.

Aguele, L.I., Omo-Ojugo, M.O. & Imhanlahimi, E.O. (2010). Effectiveness of selected teaching strategies in the remediation of process errors committed by senior secondary school students in mathematics. *Journal of Social Sciences*, 22(2), 139-144, available at: <https://www.tandfonline.com/doi/abs/10.1080/09718923.2010.11892803>.

Arhin, J. & Hokor, E. (2021). Analysis of high school students' errors in solving trigonometry problems. *Journal of Mathematics and Science Teacher*, 1(1), 1-16. DOI: <https://doi.org/10.29333/mathsciteacher/11058>.

Ashcraft, M.H. & Kirk, E.P. (2001). The relationships among working memory, math anxiety, and performance. *Journal of Experimental Psychology: General*, 130(2), 224-237. DOI: <https://doi.org/10.1037/0096-3445.130.2.224>.

Battista, M. T. (2007). The development of geometric and spatial thinking. In F. K. Lester Jr. (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 843-908). Charlotte, NC: Information Age Publishing.

Borasi, R. (1987). Exploring mathematics through the analysis of errors. *For the learning of Mathematics*, 7(3), 2-8, available at: <https://www.jstor.org/stable/40248041>.

Chamundeswari, S. (2014). Conceptual errors encountered in mathematical operations in algebra among students at the secondary level. *International Journal of Innovative Science, Engineering & Technology*, 1(8), 24-38.

Clements, D.H. & Sarama, J. (2011). Early childhood mathematics intervention. *Science*, 333(6045), 968-970. DOI: <https://doi.org/10.1126/science.1204537>.

Dawkins, P. (2006). Common math errors, available at: http://tutorial.math.lamar.edu/pdf/Common_Math_Errors.pdf.

Durkaya, M., Aksu, Z., Öçal, M.F., Şenel, E.Ö., Konyalıoğlu, A.C., Hızarcı, S. & Kaplan, A. (2011). Secondary school mathematics teachers' approaches to students' possible mistakes. *Procedia-Social and Behavioral Sciences*, 15, 2569-2573. DOI: <https://doi.org/10.1016/j.sbspro.2011.03.368>.

Gierl, M.J., Alves, C. & Majeau, R.T. (2010). Using the attribute hierarchy method to make diagnostic inferences about examinees' knowledge and skills in mathematics: An operational implementation of cognitive diagnostic assessment. *International Journal of Testing*, 10(4), 318-341. DOI: <https://doi.org/10.1080/15305058.2010.509083>.

Haghverdi, M., Semnani, A.S. & Seifi, M. (2012). The relationship between different kinds of students' errors and the knowledge required to solve mathematics word problems. *Bolema: Boletim de Educação Matemática*, 26, 649-666. DOI: <https://doi.org/10.1590/S0103-636X2012000300006>.

xe, S. (2025). How do preservice mathematics teachers analyze and respond to student errors in solving probability problems using tree diagrams? *Teaching and Teacher Education*, 168, 105250. DOI: <https://doi.org/10.1016/j.tate.2024.105250>.

Laborde, C. (2005). The hidden role of diagrams in students' construction of meaning in geometry. In J. Kilpatrick, C. Hoyles, O. Skovsmose, & P. Valero (Eds.), *Meaning in mathematics education* (pp. 159–179). New York, NY: Springer, available at: https://link.springer.com/chapter/10.1007/0-387-24040-3_9.

McGowen, M.A. & Davis, G.E. (2019). Spectral analysis of concept maps of high and low gain undergraduate mathematics students. *The Journal of Mathematical Behavior*, 55, 100686.

Mitchelmore, M.C. & White, P. (2000). Development of angle concepts by progressive abstraction and generalization. *Educational Studies in Mathematics*, 41(3), 209–238. DOI: <https://doi.org/10.1023/A:1003927811079>.

Molina, M., Rodríguez-Domingo, S., Cañadas, M.C. & Castro, E. (2017). Secondary school students' errors in the translation of algebraic statements. *International Journal of Science and Mathematics Education*, 15(6), 1137-1156. DOI: <https://doi.org/10.1007/s10763-016-9739-7>.

Newman, M.A. (1977). An analysis of sixth-grade pupils' errors on written mathematical tasks. *Victorian Institute for Educational Research Bulletin*, 39, 31–43.

Nwoke, B.I., Duru, D.C., Ahanotu, C. & Ifediba, C.P. (2024). Analyzing errors pattern in mathematics achievement among senior secondary school students: A case study. *Journal of Instructional Mathematics*, 5(1), 24-33. DOI: <https://doi.org/10.37640/jim.v5i1.1948>.

Piaget, J. (1978). *Piaget's theory of intelligence* (Vol. 2). Englewood Cliffs, NJ: Prentice Hall.

Priyani, H.A. & Ekawati, R. (2018). Error analysis of mathematical problems on TIMSS: A case of Indonesian secondary students. In *IOP Conference Series: Materials Science and Engineering*, 296(1), 012010. DOI: <https://doi.org/10.1088/1757-899X/296/1/012010>

Radatz, H. (1979). Error analysis in mathematics education. *Journal for Research in Mathematics Education*, 10(3), 163-172. DOI: <https://doi.org/10.2307/748804>.

Sapire, I., Shalem, Y., Wilson-Thompson, B. & Paulsen, R. (2016). Engaging with learners' errors when teaching mathematics. *Pythagoras*, 37(1), 1-11. DOI: <https://doi.org/10.4102/pythagoras.v37i1.348>.

Sarwadi, H.R.H. & Shahrill, M. (2014). Understanding students' mathematical errors and misconceptions: The case of year 11 repeating students. *Mathematics Education Trends and Research*, 1-10.

Scholnik, M., Kol, S. & Abarbanel, J. (2006). Constructivism in theory and in practice. In *English Teaching Forum*, 44(4), 12-20, available at: https://americanenglish.state.gov/files/ae/resource_files/06-44-4-c.pdf.

Schoenfeld, A.H. (2013). *Cognitive science and mathematics education*. Abingdon: Routledge. DOI: <https://doi.org/10.4324/9780203813924>.

Sia, C.J.L. & Lim, C.S. (2018). Cognitive diagnostic assessment: An alternative mode of assessment for learning. In *Classroom assessment in mathematics: Perspectives from around the globe*, 123-137. New York, NY: Springer International Publishing. DOI: <https://doi.org/10.1007/978-3-319-73748-5>.

Sinclair, N. & Bruce, C.D. (2015). New opportunities in geometry education at the primary school. In L. English & D. Kirshner (Eds.), *Handbook of international research in mathematics education* (3rd ed.), 319–340). Abingdon: Routledge. DOI: <https://doi.org/10.4324/9780203096143>.

Skemp, R.R. (1976). Relational understanding and instrumental understanding. *Mathematics Teaching*, 77, 20–26.

Van Hiele, P.M. (1986). *Structure and insight: A theory of mathematics education*. Cambridge, MA: Academic Press.

Veloo, A.K.H.N., Krishnasamy, H.N. & Wan Abdullah, W.S. (2015). Types of student errors in mathematical symbols, graphs and problem-solving. *Asian Social Science*, 11(15), 324-334. DOI: <https://doi.org/10.5539/ass.v11n15p324>.

Von Glasersfeld, E. (1995). Despedida de la objetividad. *El ojo del observador*, available at: <https://sotraem.izt.uam.mx/wp-content/uploads/2022/10/Glasersfeld.pdf>.